SOWISO: towards an integrated computer learning environment for mathematics

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Digital assessment-driven examples-based mathematics

with a strong online component via SOWISO, a digital environment for learning, practicing and assessing mathematics

- Digital context-rich interactive modules with theory, randomized worked-out examples and exercises
- Lecture + compulsory tutorials + digital exercising and home study
- Many short formative digital assessments
- Digital feedback in exercises is based on common mistakes known to the teacher who authored the tasks
- Support of students via forum and email
- Some SOWISO screen shots will follow to get an impression

Use of SOWISO in the Netherlands (known to me)

- UvA: at Science faculty
  basic mathematics course for biology, biomedical sciences (250 students/year)
  basic mathematics course for psychobiology (250 students/year)
  calculus for students in artificial intelligence (140 students/year)
  calculus for computer science students (70 students/year)
  remedial mathematics tutorials (500 students/year)
- VU University Amsterdam
  summercourse for prospective economy students
- Amsterdam Academic Hospital
  practical biostatistics course
- Maastricht University
  summercourse for prospective economy students (250 students/year)
  calculus course for economy students (1200 students.year)
- Leiden University
  remedial mathematics tutorials for physics students; some physics courses
- Technical University Eindhoven
  calculus summer course
- Several Dutch universities of applied sciences (Fontys, Zuyd, ..)
  mathematics courses

Screen as worksheet with feedback

Simplify the expression $\frac{4e^{7x}}{2e^{5x}}$ into the form $b \cdot e^{c \cdot x}$.

$\frac{4e^{7x}}{2e^{5x}} = 2 \cdot \frac{e^{7x}}{e^{5x}}$ Not yet in the requested form.
Did you simply copy the expression from the question or is a fraction still remaining?

$\frac{4e^{7x}}{2e^{5x}} = 2 \cdot e^{7x} \cdot e^{-5x}$ Simplify further

$\frac{4e^{7x}}{2e^{5x}} = 2 \cdot e^{7x-5x}$ OK, but not yet in the requested form

$\frac{4e^{7x}}{2e^{5x}} = 2 \cdot e^{2x}$ Okay

Repeated practice, with feedback

Examples of Instructional Materials: Working with numbers in natural sciences

from fractions to decimal notation

Convert $3\frac{2}{3}$ into decimal notation (round off to a maxumum of three decimals).

3.666	✗	You cut off to 3 decimals.
3.667	✓	Correct answer

continue > | ✖ stop | ? ask question

but with subtraction of points in score

Examples of Instructional Materials: Working with numbers in natural sciences

from fractions to decimal notation

Convert $3\frac{2}{3}$ into decimal notation (round off to a maxumum of three decimals).

3.667  Correct answer

Score: 8.0

continue > repeat stop

Repeated **worked-out** examples

Example

$10^{-2} = ?$

Answer:

$$10^{-2} = \frac{1}{10^2} = \frac{1}{100}$$

new example

Example

$(-4)^2 = ?$

Answer:

$$(-4)^2 = (-4) \times (-4) = 16$$

new example

Example

$(\frac{3}{5})^{-2} = ?$

Answer:

$$(\frac{3}{5})^{-2} = \frac{1}{(\frac{3}{5})^2} = \frac{1}{(\frac{3}{5}) \times (\frac{3}{5})} = \frac{1}{\frac{9}{25}} = \frac{25}{9}$$

new example

Stepwise practice

Applying the substitution rule

Compute:

$$\int (5y+3)^4\,dy$$

Hint

Use the substitution rule

$$\int (5y+3)^4\,dy = \_\_\_$$

Check

step-by-step

Applying the substitution rule: a.

Apply the substitution $u = 5y + 3$ to the following integral

$$\int (5y+3)^4 \, dy$$

What integral in u do you get then?

$$\int \_\_ \, du$$

Check

Solution

Applying the substitution rule: a.

Apply the substitution $u = 5y + 3$ to the following integral

$$\int (5y+3)^4 \, dy$$

What integral in u do you get then?

$\int u^4 \, du$ ✗ Wrong.Don't forget to compute the correct factor in front of the power.

$\int \frac{1}{5} u^4 \, du$ ✓ Correct answer

continue › | ✖ stop | ? ask question

Applying the substitution rule: b.

After the substituion $u = 5y + 3$ the integral $\int (5y+3)^4 \, dy$ gets replaced by the simpler integral

$$\int \frac{1}{5} u^4 \, du$$

Compute this integral

$$\int \frac{1}{5} u^4 \, du = \quad \_\_\_$$

Check

Solution

Applying the substitution rule: b.

After the substituion $u = 5y + 3$ the integral $\int (5y+3)^4 \, dy$ gets replaced by the simpler integral

$$\int \frac{1}{5} u^4 \, du$$

Compute this integral

$\int \frac{1}{5} u^4 \, du = \frac{1}{5} u^5$ ✗ Don't forget to compute the correct factor in front of the power.

$\int \frac{1}{5} u^4 \, du = \frac{1}{25} u^5$ ✓ Don't forget the constant of integration.

$\int \frac{1}{5} u^4 \, du = 0.04u^5 + c$ ✓ Correct

Questions and answers on forum

Oplossing

$$
\begin{aligned}
V(t) &= V(0) + \int_0^t V'(\tau)\,d\tau \\
&= 10 + \int_0^t 15 \cdot e^{-4\tau}\,d\tau \\
&= 10 + \left[-15 \cdot \frac{1}{4}\,e^{-4\tau}\right]_0^t \\
&= 10 + \left(-\frac{15}{4}e^{-4t} - \left(-\frac{15}{4}e^{0}\right)\right) \\
&= 10 + \frac{15}{4} - \frac{15}{4}e^{-4t} \\
&= \frac{55}{4} - \frac{15}{4}e^{-4t}
\end{aligned}
$$

Voor grote t geldt dat $V(t) \approx \frac{55}{4}\ \mu\mathrm{m}^3$ omdat de exponentiële term verwaarloosbaar klein is.
Dus is het volume van de vacuole op den duur volgens dit model gelijk aan het rechtlid, nl. $\frac{55}{4}\ \mu\mathrm{m}^3$.

0

Hoe kan 10+15/4 zo 55/4 worden?

1 month ago [redacted]

1 antwoord

0

10+15/4 = 40/4 + 15/4 = 55/4

1 month ago André Heck

Folding editor

Definition

A function of the form $f(x) = a^x$ for $a > 0, a \neq 1$ is called an **exponential function** with **base** a.

more...

Definition

We show how a^x can be defined, for positive a. If n is a positive integer, then a^n is obtained by multiplying n copies of a. In addition we define $a^0 = 1$ and $a^{-n} = \frac{1}{a^n}$. If q is a positive integer, then $b \mapsto b^q$ is an increasing function in b. For a given a there exist a number b such that $b^q = a$. We write $a^{\frac{1}{q}} = b$.
For integers p and $q > 0$, now $a^{\frac{p}{q}} = \left(a^{\frac{1}{q}}\right)^p$ is also defined. Thus for each fraction $x = \frac{p}{q}$ is a^x defined.
For general real numbers x we define a^x by continuity: if $\frac{p}{q}$ is close to x, then a^x is close to $a^{\frac{p}{q}}$.
Take for example $2^{\sqrt{3}}$. It holds $1.73205 < \sqrt{3} < 1.73206$. We see

$$2^{\frac{17}{10}} < 2^{\frac{173}{100}} < 2^{\frac{1732}{1000}} < 2^{\frac{17320}{10000}} < 2^{\frac{173205}{100000}} < 2^{\sqrt{3}}$$

and also

$$2^{\sqrt{3}} < 2^{\frac{173206}{100000}} < 2^{\frac{17321}{10000}} < 2^{\frac{1733}{1000}} < 2^{\frac{174}{100}} < 2^{\frac{18}{10}}.$$

So $2^{\sqrt{3}}$ is bounded by two already defined numbers of the form $2^{\frac{p}{q}}$ that approximate $2^{\sqrt{3}}$better and better.

less...

PDF of course notes and assessments

Question 3

What is the number of significant digits in 6.00×10^{-8}?

a. 1
b. 2
c. 3
d. 5

Theory: Computational rules: multiplication and division

When you compute with floating-point numbers you must write the outcome with the right number of significant digits and, if necessary, round off to this number of digits. The below example illustrates this.

Example

A square room has the following dimensions: 2.5×3.5 m. What is the area?
It holds: $2.5 \times 4.5 = 11.25$, but in this notation there are more siginificant digits than in any factor. The accuracy cannot increase by a computation and therefore we round off to 2 significant digits, in this case 11.
Thus, the requested area is equal to 11. m^2.

You can also interpret the above computational outcome as follows:

Because the measured size of the room is given by numbers with a precision of 1 decimal, the dimensions of the room are at a minimum of 2.4×4.4 m at a maximum of 2.6×4.6 m. Thus, the area is between $2.4 \times 4.4 = 10.56$ and $2.6 \times 4.6 = 11.96$. Taking notice of these outcomes it seems fait to round off to two significant digits.

During computations you must not round off in intermediate steps. Instead you must compute with some extra significant digits (often 1 extra) until you arive at the final result and then round off using the following general rule:

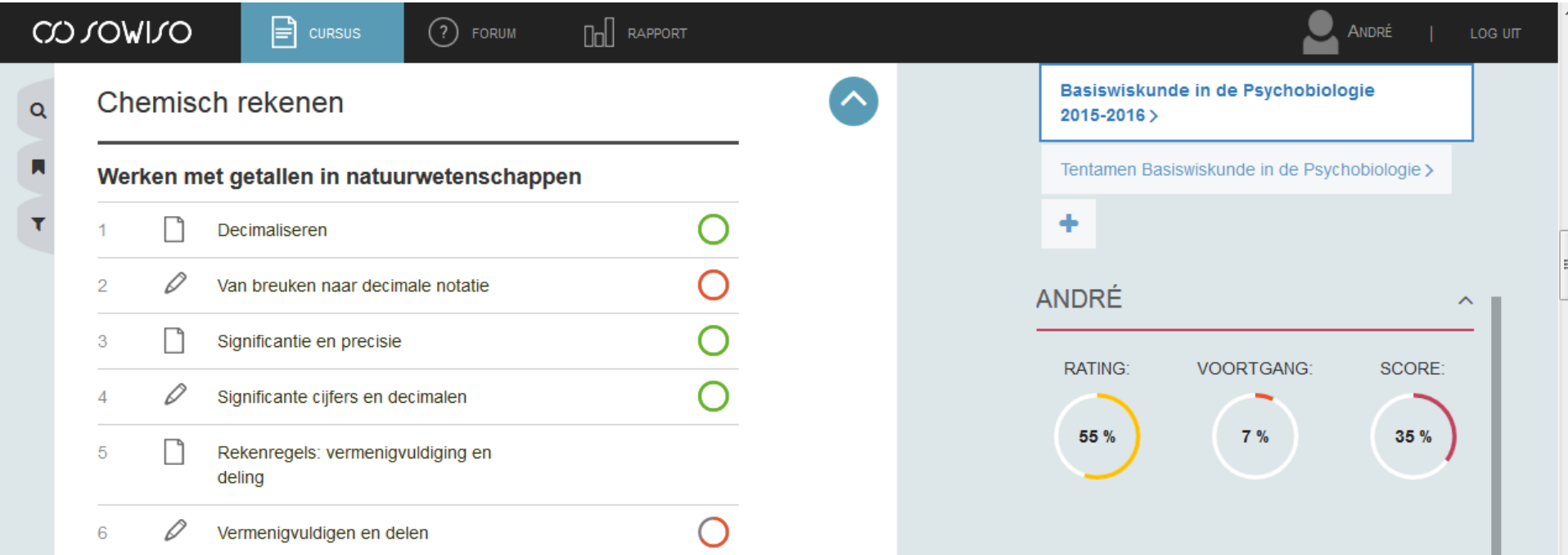
Statement

For multiplication and division holds:
The outcome of a computation has the same number of significant digits as the given quantity with the least number of significant digits.

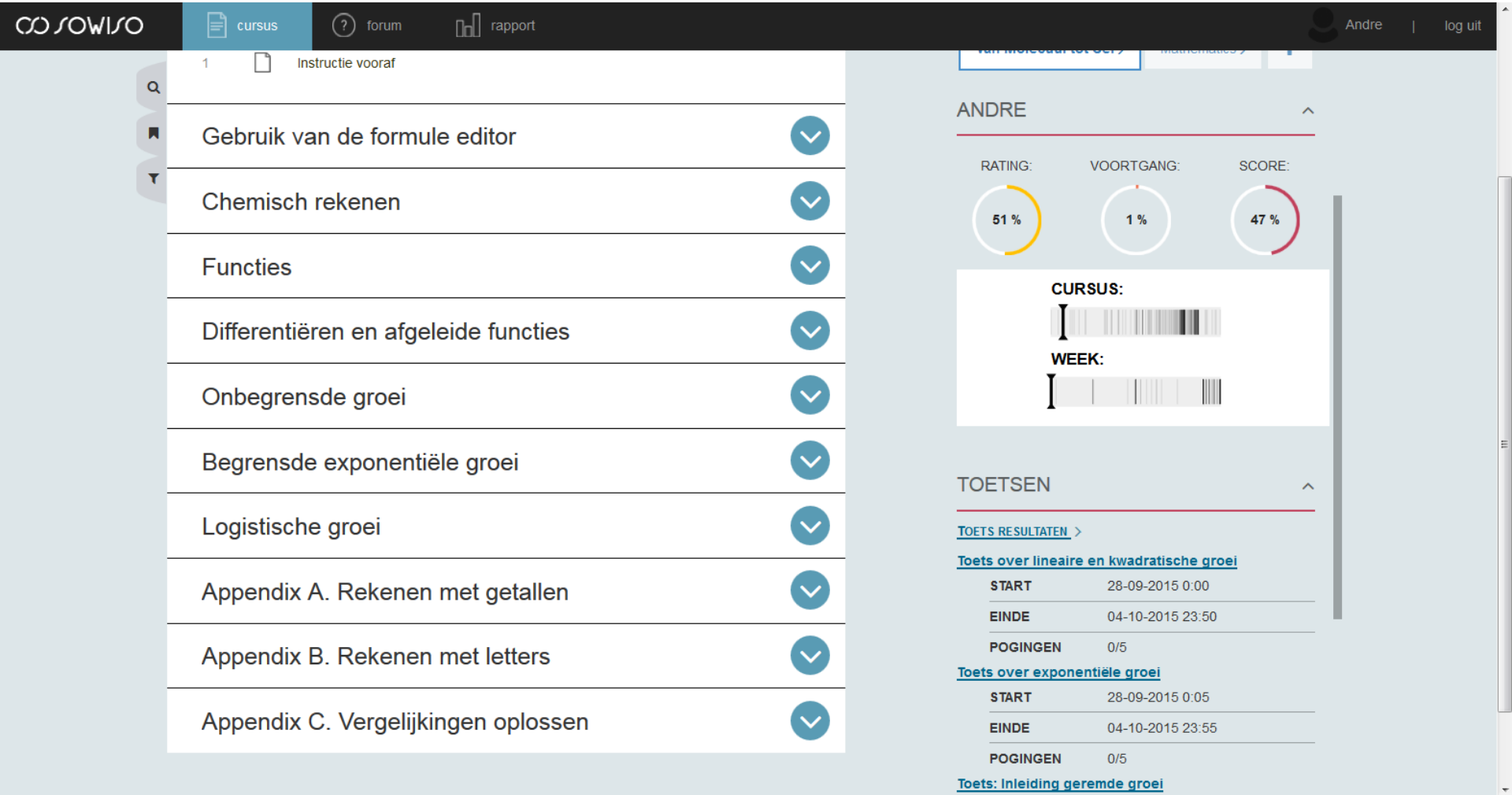
Monitoring progress in student work

Information about rating, progress and scores while working through online course

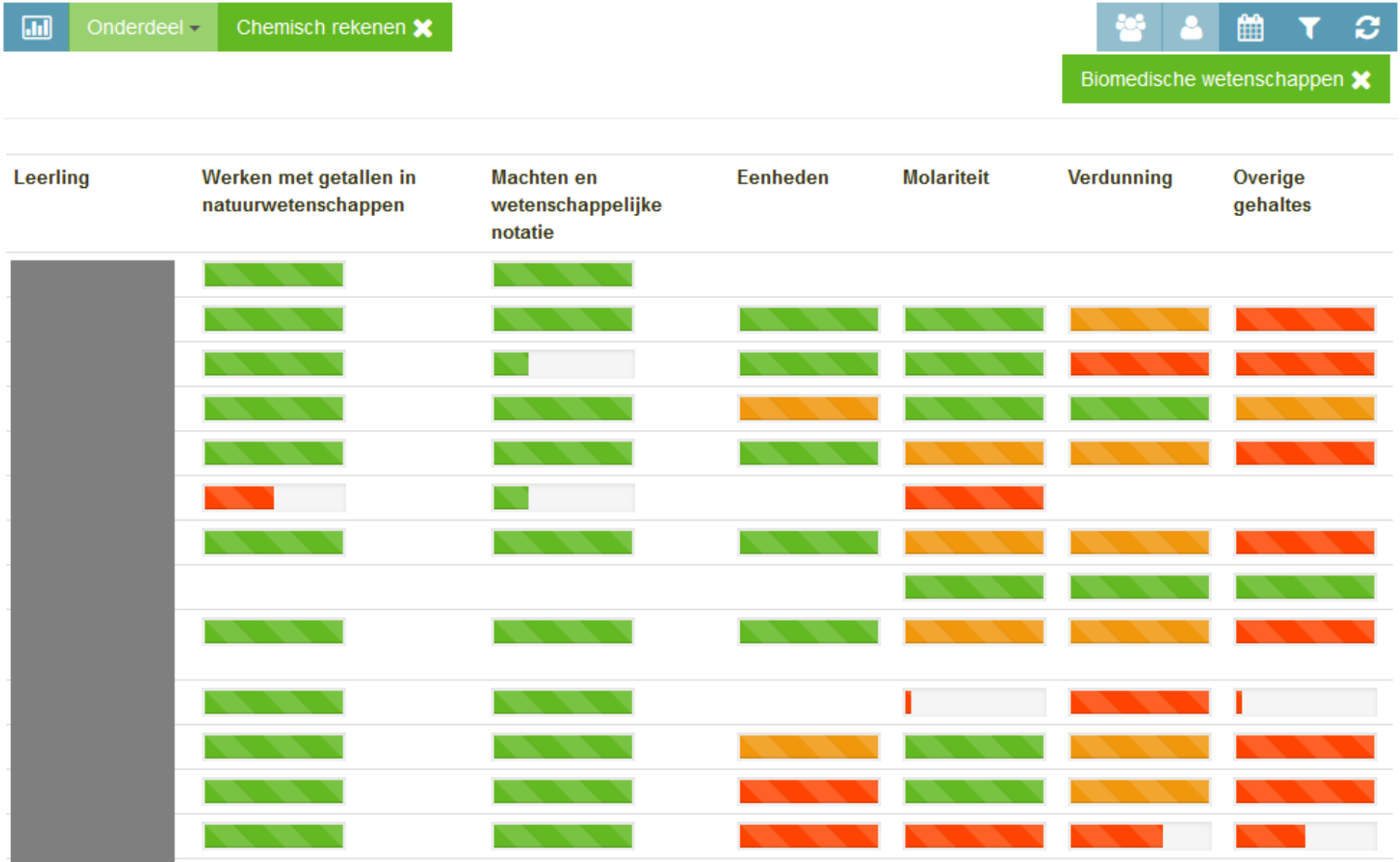
In the form of a dashboard



Other student dashboard on progress



Various progress overviews possible, for student and teacher with preferences



Reports

FILTER

Basiswiskunde in de P... | Student | 2016-02-01 / 2016-04-18 | Answered | Empty items | Filter apply

dashboard | tests | progress | activity | skills | answer analysis

most active students

Name	Activity
[redacted]	3.095
	2.836
	2.756
	2.561
	2.521

highest rated students

Name	rating
[redacted]	1.395
	1.308
	1.302
	1.247
	1.246

activity totals

Total activity	145420
Total active users	237
- Exercises finished	104.987
by number of students	237
- Theory viewed	40.433
by number of students	235

rating stats students

Name	Value
Highest rating	1.412
Lowest rating	805
Average rating	1.032
1 standard deviation	89
2 standard deviations	179

least tried exercises

Name	Count	Average
C_h1_geconjugeerde_1a	1	100
C_h1_geconjugeerde_1b	1	100

rating stats exercises

Name	Value
Highest rating	1.360
Lowest rating	259

Continuous efforts to support students via learning dashboards and help teachers analyze their courses

- VITAL Project: **Visualisation Tools and Analytics to Monitor Online Language Learning & Teaching (including the mathematics language)**
- www.project-vital.eu
- How can dashboards improve learning and teaching?

Summative CAA

Hands-On

- Login with your favourite browser at the address *uva.sowiso.nl/auth/login*

  Login: *matric1, matric2, …, matric10*
  Password: *andreheck*

- or ask me to create a personal account
  (only *name* and *email address* needed)

- Access to: *Basiswiskunde in de Psychobiologie*

Conclusions

- SOWISO environment offers a suitable e-learning environment in which students can
  - practice more than usually is done
  - get feedback on their work without teacher/tutor
  - use an interactive textbook to study
  - read the computer-generated PDF textbook
  - ask questions on the forum

and in which teachers can

  - create interactive course notes
  - monitor the students' activity level & progress level

Evaluation of ICT Mathematics

- Subject not popular for all, but doable for all
- No postponement behaviour (students work!)
- Fewer mistakes during laboratory work with mathematical calculations
  (think of diluting, pH calculation, etc.)
- Students have a need for or just like
  - explanations from teacher, tutors, and peers
  - worked-out examples
  - contact time
- Digital environment supports learning

Major challenges for an author using a CAS-based tool

- Author must be familiar with CAS
- Intelligent feedback requires rather sophisticated ‘programming’
- Difficult to foresee the construction of an unsolvable or trivial problem when algorithmic parameters come into play
- Creation of good randomized exercises costs time (my average is 2 hours per exercise)
- Exercises for development of procedural knowledge and skills are doable,
  but what about exercises to develop problem solving skills and conceptual knowledge?

Key issues for success of CAA

- Flexible authoring
- Algorithmic parameters
- Intelligent & immediate feedback
- Integration with VLE and administration system
- Functionality in practice

And they are all equally important!

Advice for people new in the field

Start simple:
- Try first non-randomized versions
- Copy existing exercises and change them
- Gather good examples for reference
- Start first with obvious randomizations

Learn from other people
- copy good constructs

Document methods that you discover

Literature

André Heck & Natasa Brouwer (2015). Digital assessment-driven examples-based mathematics for computer science students. In N. Amado & S. Carreira (eds.) *Proceedings of the 12th International Conference on Technology in Mathematics Teaching* (pp. 403-411), University of Algarve, Faro, Portugal. ISBN 978-989-8472-68-7; Available at http://hdl.handle.net/10400.1/6081