# Bin that rubric
# A better way to assess conceptual understanding

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EAMS 2016, Newcastle-upon-Tyne

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Mathematics Education Centre

# The Module

- Foundation Mathematics Part II
- Polynomials, Partial Fractions, Matrices, Complex Numbers, Vectors, Integration, Statistics
- 140 students, C at GCSE to A at A level
- Assessment:
  Exam 80%, STACK 10%; Conceptual test 10%

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How would you explain to someone what a **derivative** is?

Use examples, diagrams and writing to help make it clear. Include everything you know about derivatives. Write only in the box.

fundamentally a derivitive is a product of another source. it is the son to a mother if you like. it can be applied to many different areas of mathematics but will always produce a product. to derive is to arrive via instruction. using $\frac{dx}{dy}$ we differenciate and here for derive the product or derivative.

$$3x^2 + 4x + 3 \to \frac{dx}{dy} = 6x + 4$$

it can also be used in other examples more simple such as to find the derivitive of

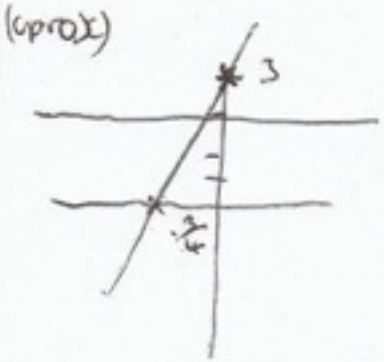
$3x + 4$ when $x = 2$ $\quad 3x + 4 \to = 10$

a general rule for deriving form vers is

$$y = x^n \to y = nx^{n-1}$$

this can also be solved graphically

(aprox)




$3x^2 + 4$ $\qquad \frac{dy}{dx} = 3$

# Student instructions

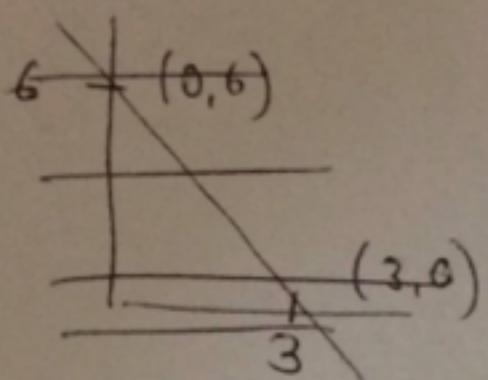
- Photograph or scan your test
- Do not include your name
- Upload to VLE (Moodle)
- Receive judging link
- https://www.nomoremarking.com/j/K9tsYV

# Try it yourself

tinyurl.com/EAMS130916

Left | Better understanding of derivatives? | 3 | Right

A derivative is used ~~a more accurate gradient of a line~~ to find the ~~exact point~~ gradient on a line.




~~You would assume the gradient is $\frac{dy}{dx} = \frac{6}{3} = 2$.~~

~~However~~ if the line of a graph is $y = 6x + 2$ $\therefore$ $x = 3$, $y = 20$

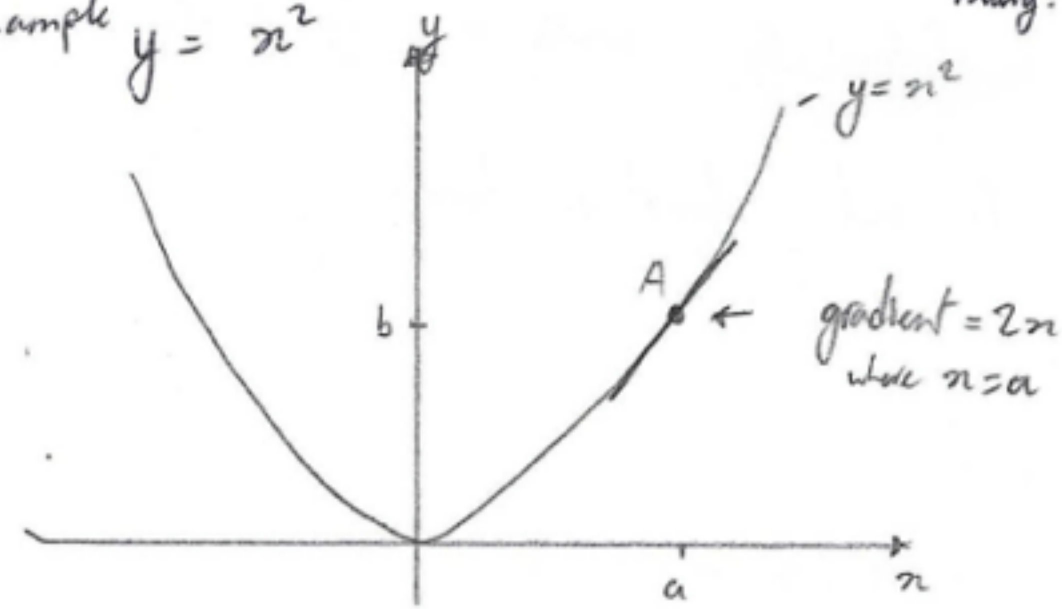
$$\frac{dy}{dx}(6x^2 + 2x) = (12x + 2)^{x=3}$$

When $x = 3$, $y = 38$

The principle follows the rule of $\frac{dy}{dx}$ of $n^x = xn^{x-1}$.

A derivative is a technique used to find the gradient of the equations' line/curve, at any point on the curve. $\boxed{\frac{dy}{dx} = nx^{n-1}}$ = general rule, although there are many.

Example $y = x^2$




to find the gradient at point a, find the derivative of $x^2$. $\frac{dy}{dx} = 2x$

then input a instead of x to find the gradient.

This is useful in the real world to find the rate of flow of water through a pipe for example.

# Student incentive

Do you want peer feedback on your test? If so then you must (1) **complete at least 10 judgements** (more is fine!) and (2) **provide some feedback** about your judging. Instructions on how to provide feedback are below. It is a good idea to familiarise yourself with the below four feedback questions before starting the judging. Then you'll know what to think about when doing the judging.

You should supply one or two sentences for each of the following questions. So your response will contain between four and eight sentences in total.

1. How did you decide which tests were better?
2. What does a typical good answer look like?
3. What does a typical not-so-good answer look like?
4. Was this valuable for learning about what makes a good answer?

# Student incentive

- Rehearsal for two real assessments (test + judging) later in semester
- 140 students:
  - 51 uploaded a test
  - 31 judged the tests
    (range 1 to 151, median = 13)
  - 14 provided peer feedback

# Peer feedback

For choosing which test was the better one, it had to contain a few key features such as a clear answer with a somewhat neat presentation. It also had to have a basic definition of a derivative written in two or three sentences, backed up with illustrations or an example of how to find a derivative and what are its uses.

A not-so-good answer would lack these key features or contained just one of them.

This helped see other people answers, mentioning information that I didn't think about and formulating the answer in a way that could improve mine.

1. Looked for explanation of the concept before diving in to examples. Rated more highly concise explanations that did not include unnecessary information on, for example, maxima and minima.

2. Short textual explanation, simple worked example and diagram of typical usage with graphs/gradients.

3. Too wordy, contained unnecessary information, did not explain derivatives conceptually, clunky writing.

4. Useful to see how people managed to explain the same thing with far more brevity than I did.

# Lecturer feedback

**STUDENT #1**
You don't provide a specific definition of derivative, which is the function that gives the gradient at any point of another function.
Your example could have explained better what is going on, e.g. explaining differentiation from first principles, or providing the general rule for differentiating a polynomial.
Perhaps write in ink not pencil, it's a bit faint.

**STUDENT #2**
Your test was submitted late and not included in the judging. Make sure it is on time in furture!
You provided a good definition in written English.
Your graph could have included a tangent line and explanation of how to calculate it.
More use of mathematical notation and examples of derivatives would have strengthened your answer.

**STUDENT #3**
You don't provide a specific definition of derivative, which is the function that gives the gradient at any point of another function.
It's differentiation, best not to mention integration as it may make people think you are unsure.
I liked the rules for differentiation, although fewer rules explained in more detail might have worked better.
Sorry you ran out of time! I'm sure you'll get the hang of maximising the ten minutes allowed in the future.

**STUDENT #4**
Your photo was a bit out of focus, make it sharper next time.
You provide an explanation, examples of notation, and a graph with a tangent - all very good.
However I think more explanation would have helped make an overall more coherent piece of work. For example, nice to include the

# Diagnostic test 2

In mathematics we often transform an expression into an equivalent form that involves only **polynomials**. This might involve, for example, expanding brackets, using the Maclauren series, or using partial fractions.

Explain why it can be useful to do this. You can use examples, symbols, diagrams and writing to help explain your understanding.

1)

- LINEAR EXPRESSION = $x + 1$
  RAISED TO THE FIRST POWER.
- POLYNOMIAL EXPRESSIONS
  RAISED TO A POWER BEYOND THE FIRST.

$$\left.\begin{array}{lll} \text{i) QUADRATIC} & = & x^2 + 1 \\ \text{ii) CUBIC} & = & x^3 + 1 \\ \text{iii) QUARTIC} & = & x^4 + 1 \\ \text{iv) QUINTIC} & = & x^5 + 1 \end{array}\right\}$$

THE POWER TO WHICH $x$ IS RAISED TO, IS KNOWN AS THE DEGREE.

2) TRANSFORMING EXPRESSIONS:

i) EXPANDING BRACKETS: TWO TERMS ENCLOSED IN THEIR RESPECTIVE BRACKETS SUGGESTS FACTORISATION. THIS COMPACT WAY OF REDUCING POLYNOMIAL EXPRESSIONS CAN BE EXPANDED BY MULTIPLYING EACH TERM IN ONE BRACKET BY THE OTHER.

ii) ~~TAYLOR MACLAURIN SERIES~~: USED AS PARTIAL FRACTIONS

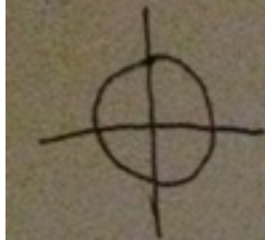
A TECHNIQUE TO SEPERATE DIFFICULT POLYNOMIAL ALGEBRAIC FRACTIONS INTO MORE MANAGABLE ONES.

— TIME UP.

# Diagnostic test 3

Explain everything you understand about the **mathematics of circles**.

Write only in the box.



Equation of a circle is $(x-a)^2+(y-b)^2=r^2$
where $r$ is the radius of the circle and $(a,b)$ is the centre of the circle

~~Differentiation~~ Differentiating the equation of a circle the gradient of a line that is a tangent to ~~on~~ the edge of a circle passing through a coordinate is found.

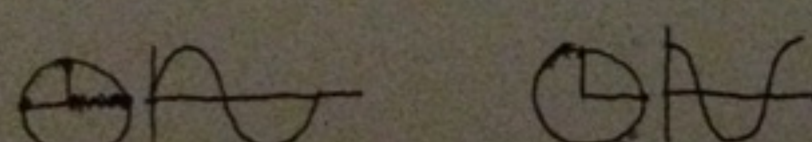
~~That circle is ~~a~~ formed from horizontal plane that intersects a double cone~~

A circle is formed from the intersection of a horizontal plane through a double cone

Circles have perfect symmetry, ~~thus~~ the radius is constant at every angle around the ~~cre~~ circle.

The circle is 360° or $2\pi$ radians

A phasor going around a circle at 90° or $\pi/2$ intervals at different phase angles produces the sine and cosine graphs



# Class tests

**Class tests**

The class tests are open-ended tests that have no right or wrong answers. They are designed to test how well you understand the underlying concepts of the module, and how well you can communicate clear mathematical thinking. Understanding, reasoning and creativity are important to get a good mark.

- **Test 1** contains one question and it might be drawn from any of the following topics: polynomials, partial fractions  matrices, conic sections and polar coordinates (i.e. Chapters 1 to 5). You will be given 15 minutes to answer the question. The test will take place in the lecture on Friday 11 March.
- **Test 2** contains one question and it might be drawn from any of the following topics: complex numbers, vectors, further integration, applications of integration and probability (i.e. Chapters 6 to 10). You will be given 15 minutes to answer the question. The test will take place in the lecture on Friday 6 May

Following the tests you will have one week to complete a peer assessment exercise online. This will involve making judgements about the quality of other students' work. We will practice these types of questions and judging what makes a good quality answer during the module.

Each class test counts towards 5% of your module mark. This breaks down as 4% for the quality of your answer to the test question, and 1% for the quality of your judgements of other students' answers.

# Class tests

## Judging Class Test 1 instructions

You will receive your link by automated email to your student account on the afternoon of Tuesday 15 March.

You are required to make at least 19 judgements. The deadline for completing your judgements is 9am Tuesday 22 March. It is advised you do not use Internet Explorer, choose another browser instead.

You will receive a score out of 5 for your class test. 4 marks are for the quality of your answer, and 1 mark is for completing your judgements.

Email Ian if you have any problems or questions about this.

# Class tests

What is meant by the **determinant of a matrix**?

Use symbols and writing, and give examples of how determinants can be useful, to help explain your understanding.

How would you explain to someone what a **complex number** is?

Use examples, diagrams and writing to help make it clear. Include everything you know about complex numbers. Write only in the box.

A matrix is an ~~series~~ array displayed within rows and Columns, for example:

$$\begin{bmatrix} 3 & 6 \\ 7 & 9 \end{bmatrix}$$

A Square matrix is a matrix where the number of rows is equal to the number of columns.

A matrix can only have a determinant if it is Square, the determinant can then be used to find the inverse matrix as long as the determinant is not zero, When a determinant is equal to zero it is known as a singular matrix as it has a single solution.

The determinant of a matrix (the matrix being:

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

in this example) ~~is~~ is given the notation "$|A|$" similar to the notation for modulus. The determinant for a square matrix with order 2 can be calculated as:

$$|A| = ad - bc$$

Then the inverse of the matrix can be found using the determinant as shown below:

$$A^{-1} = \frac{1}{|A|} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

The inverse matrix in turn can be used to work out Simultaneous equations.

A Complex number involves the strange situation of a ~~Strange~~ Square number being negative.
·To this day imaginary numbers are unknown but are useful. An example of an imaginary number is $j^2=-1$.
Complex Numbers are useful in quadratics such as "$x^2+10x-40=0$" - which the solution is $5\pm\sqrt{-15}$.
Commonly used in real World and engineering.

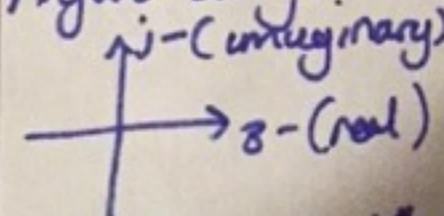
$j=\sqrt{-1}$.

Evaluating:
$j^6=j^2\times j^2\times j^2 = -1\times-1\times-1=-1$

$\frac{1}{j}=\frac{1}{j}\times\frac{j}{j}=\frac{j}{j^2}=\frac{j}{-1}=-j$

Two letters are Commonly used: "i" and "j". Engineers use "j".

A Complex Number is made from an imaginary part and a real part. with a general form of $z=a\pm jb$. where $a$=real number and $jb$ = Complex Number. On an Argand diagram you can plot a complex number:

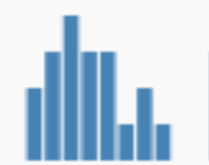


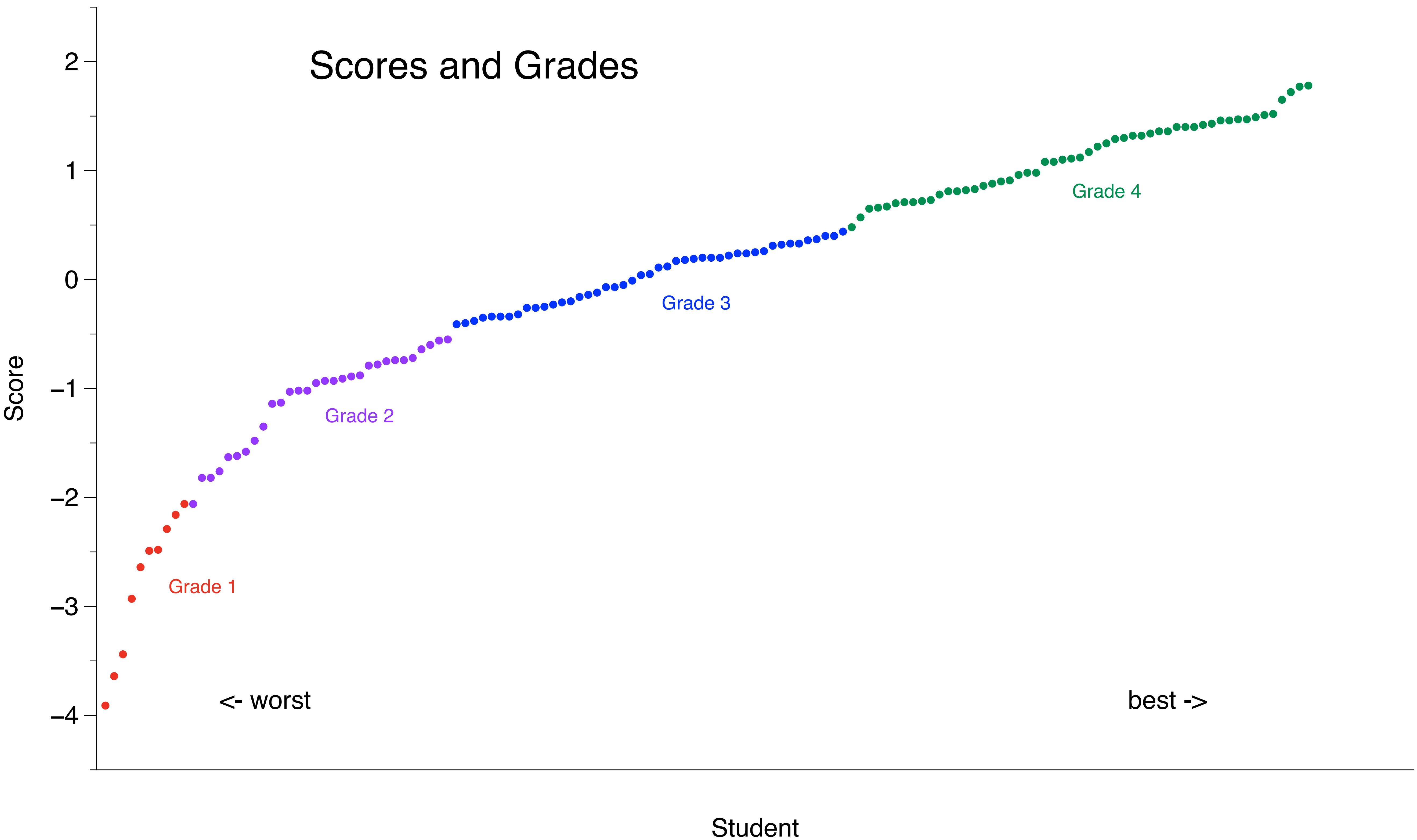

A Complex Conjugate is the Sign reversed, usually defined as $z^*$ or $\bar{z}$. Complex Numbers can be multiplied and divided algebraically. Also be ~~multiplied and~~ added and subtracted. ~~For~~ Complex numbers also useful with AC circuits, finding total Voltage.
Complex Numbers can be defined in Polar form, as $a=r\cos\theta$ and $b=r\sin\theta$. Finding the modulus is $\sqrt{a^2+b^2}=|z|$ with polar Numbers you can find the argument (angle) and modulus (length). De Moivres theorem allows you work with rational numbers, therefore can be used to Calculate positive powers:
$z^n=r^n(\cos n\theta \pm j\sin\theta n)$

# Matrices test

- 138 uploaded a test
- 86 completed >19 judgements (median 21)
- 155 moderation judgements
- Score generated for each student
  - Internal consistency = 0.94
  - Inter-rater reliability > 0.88
  - Fit statistics good

# Judging quality

| | Infit | Judgements | Median Time | % Left Clicks | Created At | Time Chart |
|---|---|---|---|---|---|---|
| | 1.12 | 22 | 0:00:10 | 36.36% | 15/03/16 10:29:32 |  |
| | 1.05 | 22 | 0:01:13 | 45.45% | 15/03/16 10:29:32 |  |
| | 0.73 | 22 | 0:00:23 | 45.45% | 15/03/16 10:29:32 |  |
| | 0.91 | 22 | 0:01:09 | 36.36% | 15/03/16 10:29:32 |  |
| uk | 0.87 | 21 | 0:01:24 | 38.10% | 27/03/16 23:29:45 |  |
| | 0.89 | 21 | 0:00:17 | 52.38% | 15/03/16 10:29:32 |  |
| | 1.09 | 21 | 0:00:21 | 42.86% | 15/03/16 10:29:32 |  |
| | 1.22 | 21 | 0:00:11 | 9.52% | 15/03/16 10:29:32 |  |


Scores and Grades
Score
Student
2
1
0
−1
−2
−3
−4
Grade 1
Grade 2
Grade 3
Grade 4
<- worst
best ->

## FEEDBACK

The following feedback describes the overall features of answers that scored between 0 and 4 for the answer part of the test. Remember that an additional mark of 0 or 1 was awarded for undertaking the judging. The below descriptors are broad and general, it does not mean that every single answer fits neatly into the categories as described below. Indeed, there was a great range of approaches and types of answer as you will have seen when completing your judgements. Nevertheless, the below catches overall themes in the answers at each score.

**SCORE 0**
Unsubmitted answers scored 0.

**SCORE 1**
Answers that scored 1 were partial and sometimes not so clearly explained. A typical answer would state what a determinant is and perhaps give an example, but not explain how determinants can be used to solve problems. Explanations were sometimes very formal, without much additional writing or the use of examples to help the reader understand.

**SCORE 2**
Answers that scored 2 addressed both parts of the question, but did so quite minimally. Typical answers defined determinants and gave an example how to calculate a determinant with varying degrees of detail and clarity. Most answers at score 2 gave just one example of how determinants can be useful, often explaining how to determine if a matrix is singular, or showing how to calculate an inverse matrix. Sometimes explanations or examples might have been clearer.

**SCORE 3**
Answers that scored 3 were reasonably comprehensive and well explained. A typical answer would define clearly what is meant by a determinant, and give examples too. Writing and formal mathematics were commonly employed in explanations. Many of the answers that scored 3 gave one or two good examples of how determinants can be used to calculate an inverse matrix, and some answers mentioned solving simultaneous equations although a mathematical example was not always provided.

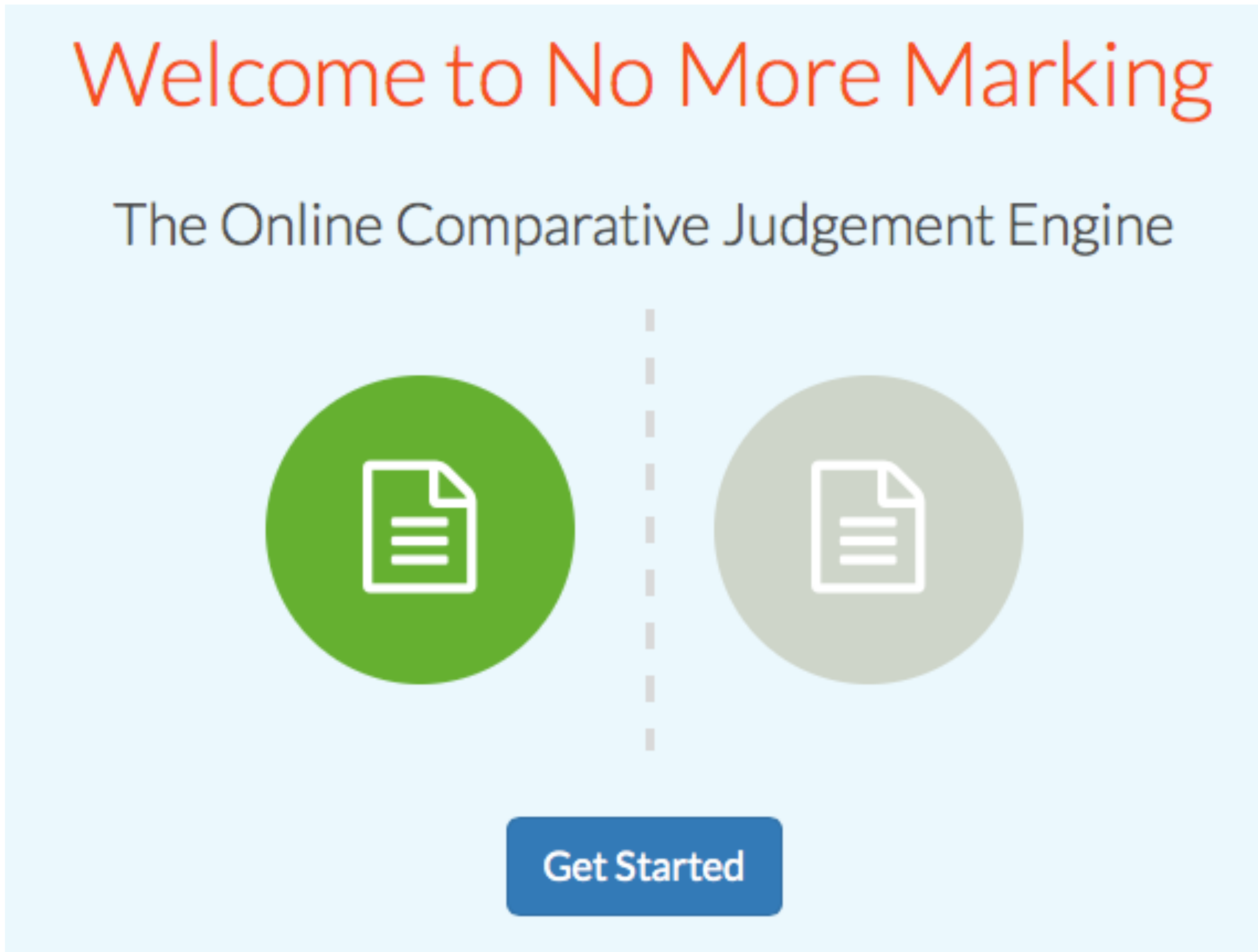
**SCORE 4**
Answers that scored 4 were comprehensive and clearly explained. A typical answer would define clearly what is meant by a determinant, and give examples too. Often writing and formal mathematics were used to help to do this. Most of the answers that scored 4 gave one or more detailed and correct examples of how determinants can be used to solve systems of simultaneous equations with the matrix method. Answers also often included stepwise explanations of inverse matrices and how examples of how they can be calculated.

# Research background

- Undergraduate calculus, statistics
- Secondary algebra, calculus, fractions
- Primary algebra
- Conceptual rather than procedural

Bisson, Gilmore, Inglis & Jones (2016). *IJRUME, 2*, 141-164.
Jones & Karadeniz (2016). *PME 40, Vol. 3*, 51-58.
Jones & Wheadon (2015). *SEE, 47*, 93-101.
Jones & Alcock (2014). SHE, 39, 1774-1787.
Jones, Inglis, Gilmore & Hodgen (2013). *PME 37, Vol. 3*, 113-120.

# Try it yourself

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# Thank you

**Ian Jones**
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