

Mathematica-based e-Learning/Assessment System for College Mathematics

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- Effectiveness

E-A×M+S 2016

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What is 'MATH ON WEB' ?



Summary of 'MATH ON WEB'

- Target : College Mathematics (Linear Algebra and Calculus)
- Consists of 2 systems: WMLS and WASM

WMLS : Web-based Mathematics Learning System

WASM : Web-based Assessment System of Mathematics

- Background CAS = web*Mathematica and Mathematica* (Wolfram)
- Used for after-class learning
- More than 1200 questions implemented



Search "MATH ON WEB"...



約 15,000,000 件 (0.27 秒)

<u>大阪府立大学 | MATH ON WEB Learning College Mathematics by ...</u>

www.las.osakafu-u.ac.jp/lecture/math/MathOnWeb/ -

大学初年次の数学に関する「計算ドリル型教材」と「シミュレーション型教材」が利用できます。 大 阪府立大学生はこちらから・その他の利用者はこちらから. 数学到達度評価システム. 大学数学の学習内 容の到達度をオンラインテストで評価します。 大阪府立 ...

WebMath - Solve Your Math Problem www.webmath.com/ マこのページを訳す

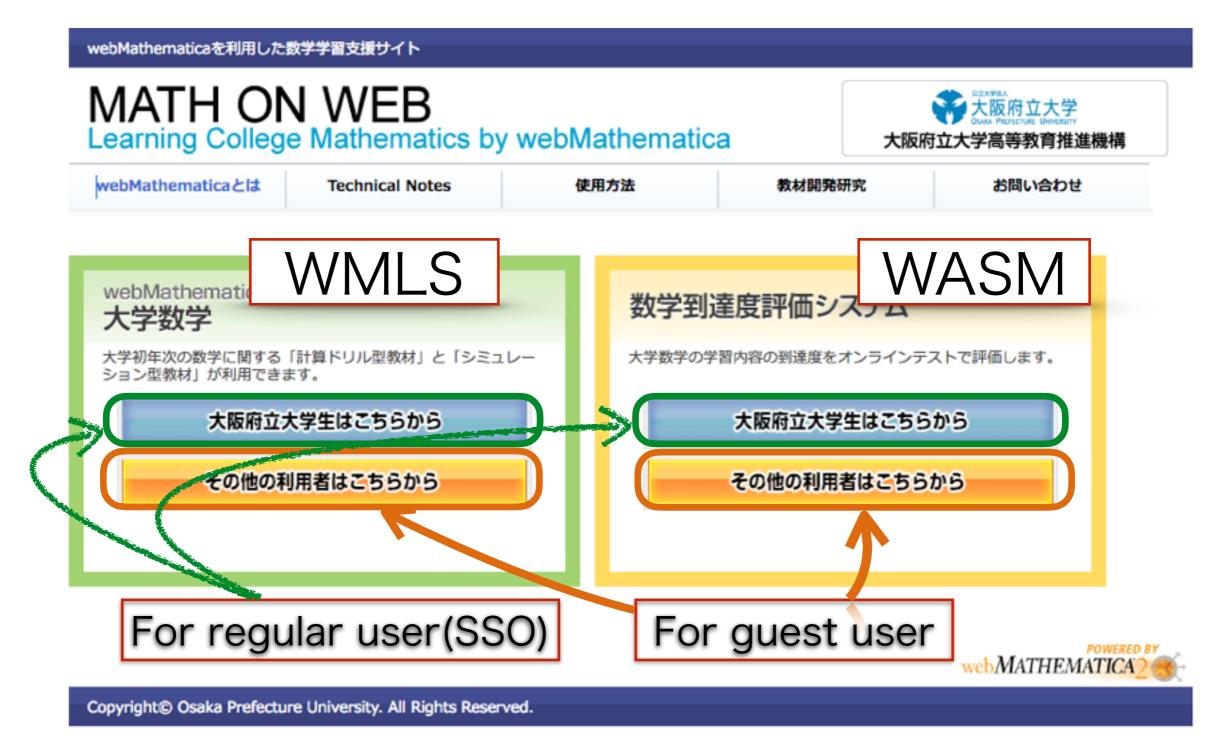
Webmath is a math-help web site that generates answers to specific math questions and problems, as entered by a user, at any particular moment. The math answers are generated and displayed real-time,

http://www.las.osakafu-u.ac.jp/lecture/math/MathOnWeb/



Screenshots

http://www.las.osakafu-u.ac.jp/lecture/math/MathOnWeb/



Screenshots (WMLS)



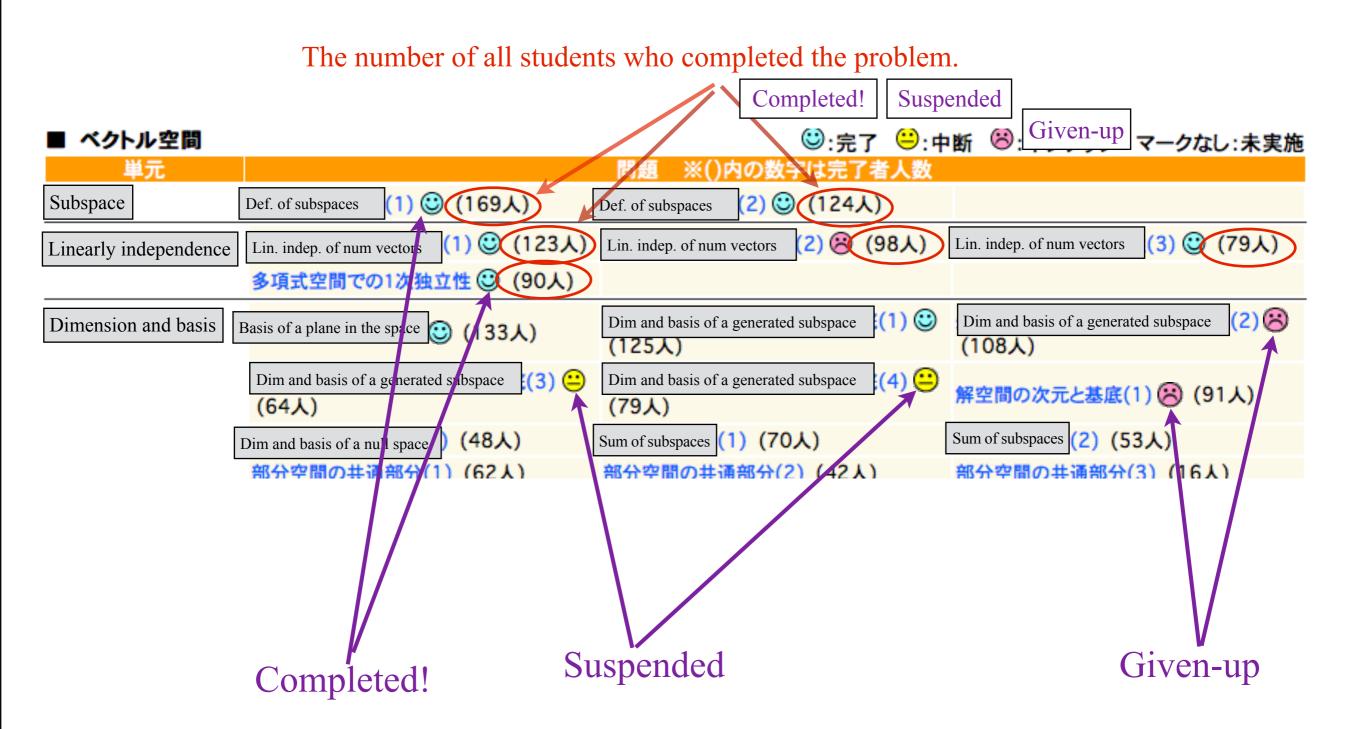


List of Learning Contents (WMLS)

Vector space			
単元		問題	
Subspace	Def. of Subspaces(1)	部分空間の定義(2)	
Linearly independency	Lin. ind'cy of num vectors (1) Lin. ind'cy of polynomials	数ベクトルの1次独立性(2)	空間の次元と基底(1) Problems list
Dimension and Basis	空間内の平面の基底	生成される部分空間の次元と表	
	生成される部分空間の次元と基底(3) 解空間の次元と基底(2) 部分空間の共通部分(1)	生成される部分空間の次元と基 部分空間の和(1) 部分空間の共通部分(2) 例题	問1. $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$
3 元斉次連立1 次 問1.			Image: B12. A =
S $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$ に対し、Vを連立一次方程式Ax=0の解全体のなす R ³ の部分ベクトル空間 Vの次元と基底を求めよ。			B3. $A = \begin{pmatrix} 1 & 1 & -1 \\ 2 & 2 & -2 \\ -1 & -1 & 1 \end{pmatrix}$
解答欄: 次	x元は, 基底は{), (), ()}	Image: Height Heigh
	するベクトルが3個より少ないときは、余った ときは,基底の欄は空欄にすること。	ベクトルの欄は空欄に すること。	Image: Big 1 0 1 1 1min 2 -1 1 1

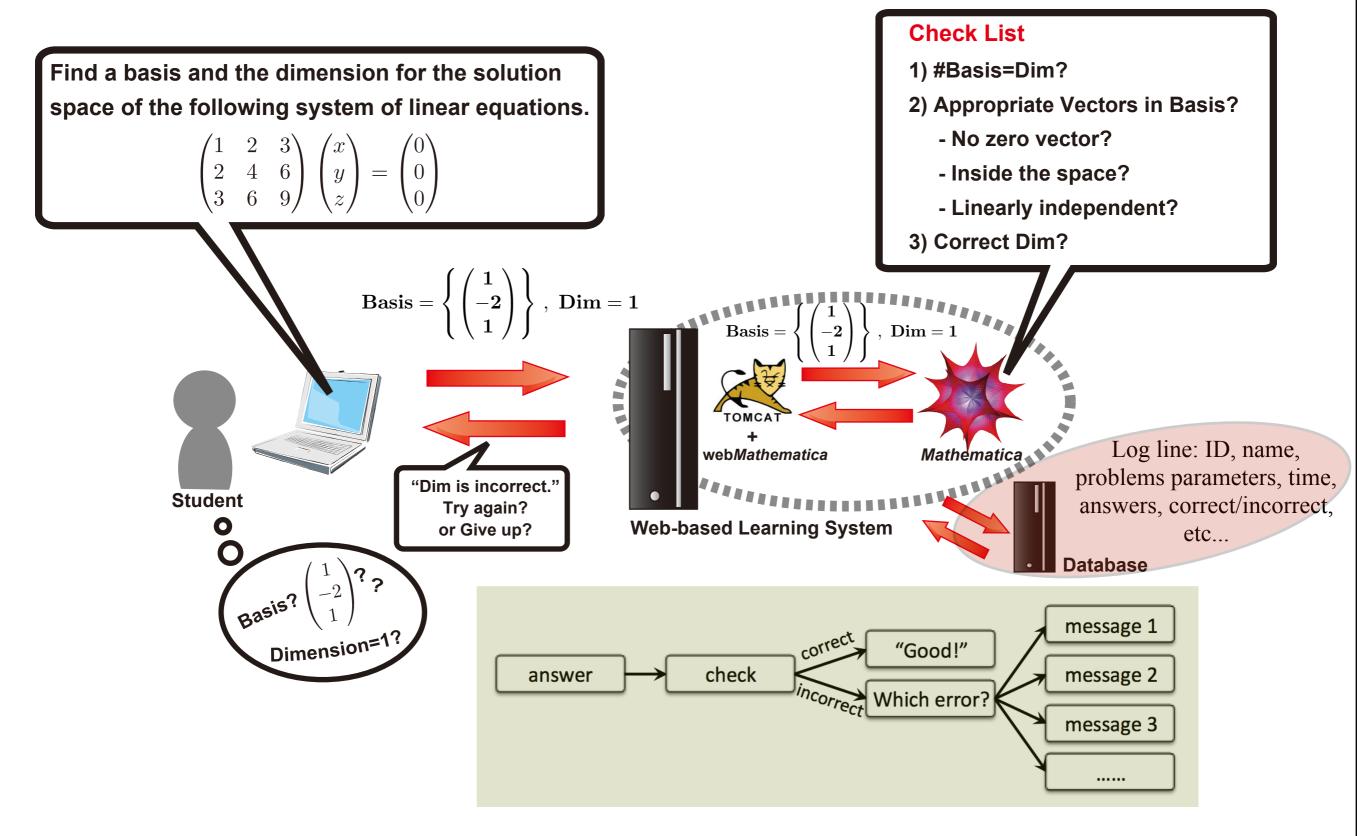


Feedback of learning status (WMLS)





The system summary



Features of WMLS and WASM

WMLS Key feature:

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- WMLS is a self-learning system.
- WMLS consists of the drill section and the simulation section.
- The system has over 200 learning units and each unit consists of 5 problems. (Over 1000 problems!)
- Each learning unit is equipped with a problem example and its model answer in PDF format.
- There is no assessment mode, no time limit.
- Students can retry the same problems repeatedly until they get correct answers.
- The correct answer is never shown.

WASM Key feature:

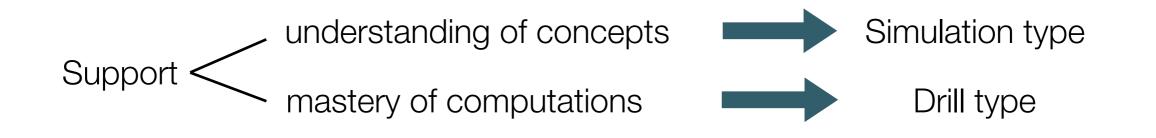
- WASM is an online assessment system.
- Assessment mode and drill mode have been implemented.
- Assessment materials are associated to the learning units.
- Problems are randomly generated.
- Answer time limits are set in all problems in an assessment mode.
- The correct answer is never shown.
- No simulation section.

Motivation

To develop the students' understanding of the students, we want to support and promote after-class learning.

To support after-class learning, we thought web-based system might be appropriate.

We wondered how to develop a web-based simulation system, web*Mathematica* 1.0 was launched timely (2002).



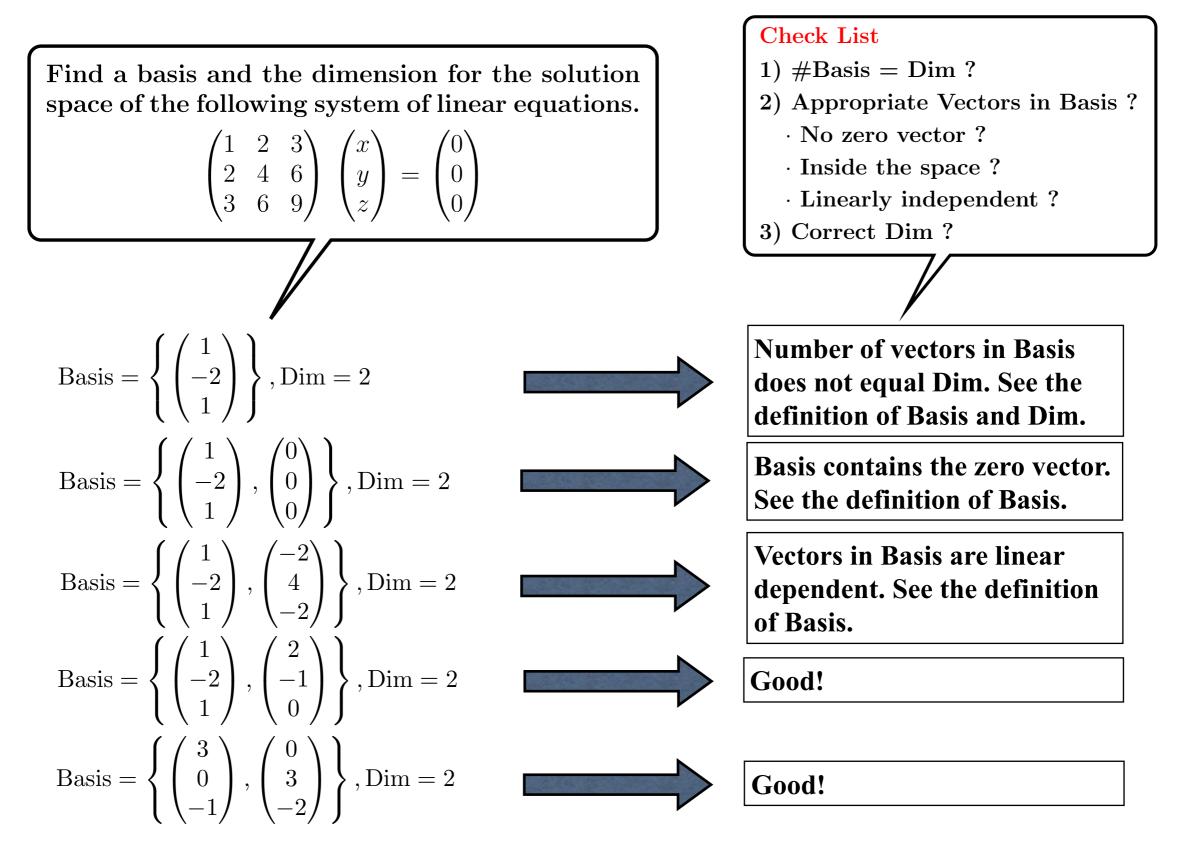


The merits of the drill system

- Paper-based exercise ... students want to know the answer.
- There are some questions which have non-unique answer ... CAS can handle such cases with appropriate program.
- The program not only can judge the correctness, but also can respond appropriate feedback message to the students.



Feedback samples





Project and related history

- 2002 pre-WMLS(Simulation) was developed with web*Mathematica* 1.0
- 2003 pre-WMLS(Drill type) was developed with web*Mathematica* 1.0
- 2005 STACK 1.0 was launched.
- 2005 Practical use of pre-WMLS for all students started.
- 2009 WMLS was developed with support by MEXT(2009-2011).
- 2012 WASM was developed with support by MEXT(2012-2013).
- 2013 Tentative use of WASM started.
- 2014 Practical use of WASM started.
- Oct 2014 STACK 3.x became available in our university!!

MEXT : Ministry of Eduction, Culture, Sports, Science and Technology, Japan

Practical use of the systems



Learning environment with WMLS

We use WMLS :

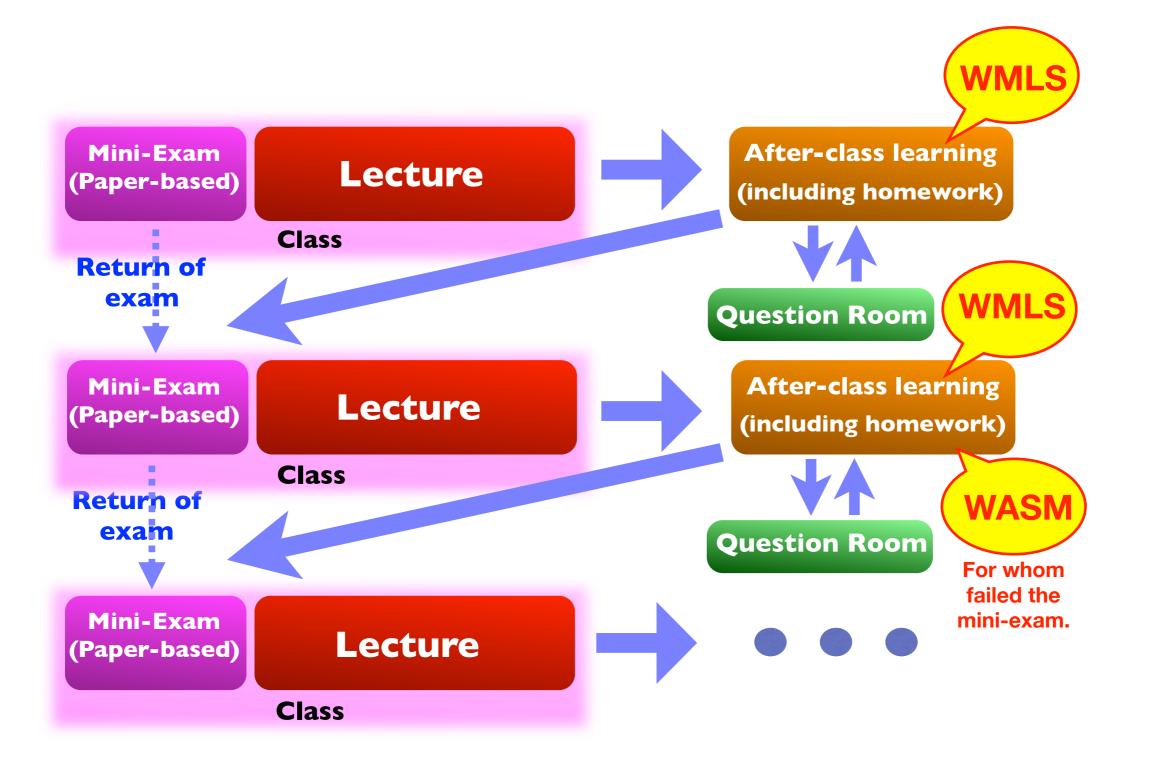
- 1. In all the classes of linear algebra and calculus.
- 2. As a tool for after-class learning.
- 3. To complement the regular class activities.
- 4. Combined with public office hour (in a designated room(*))

Teachers not compel but recommend the students to use the system.

(*) We call this room "*Question Room*", opened regularly in the semesters afternoon.



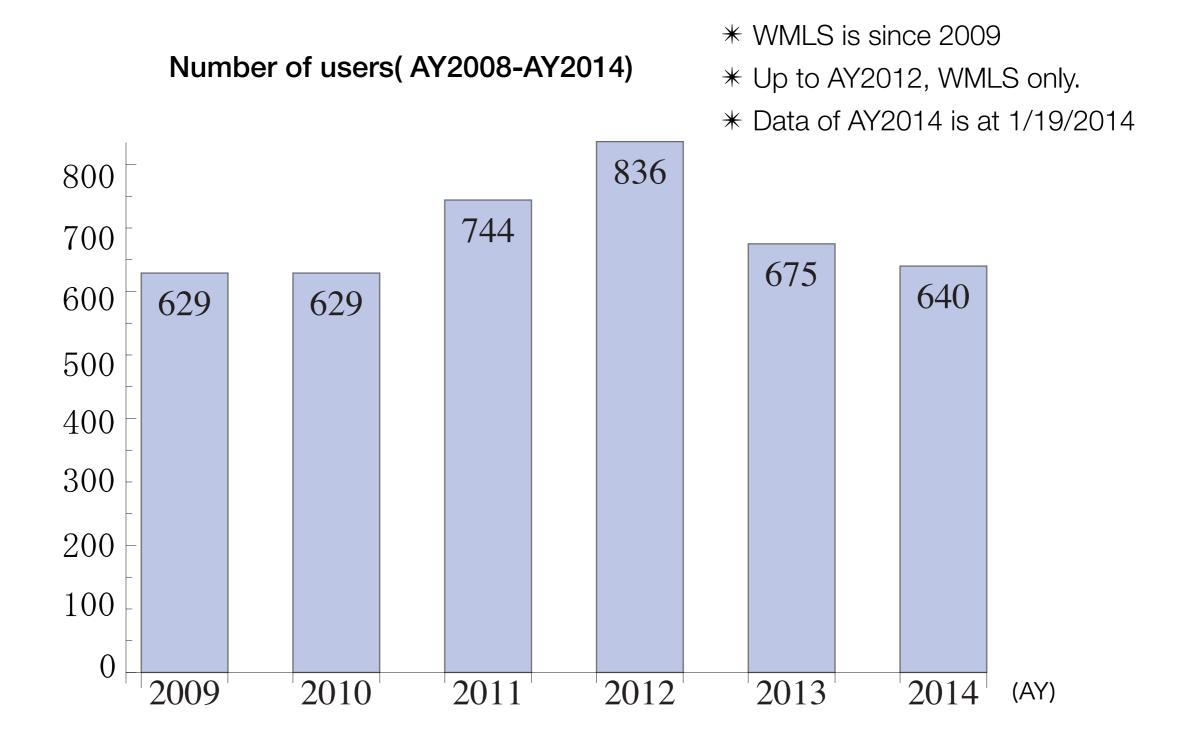
Example class with 'MATH ON WEB'



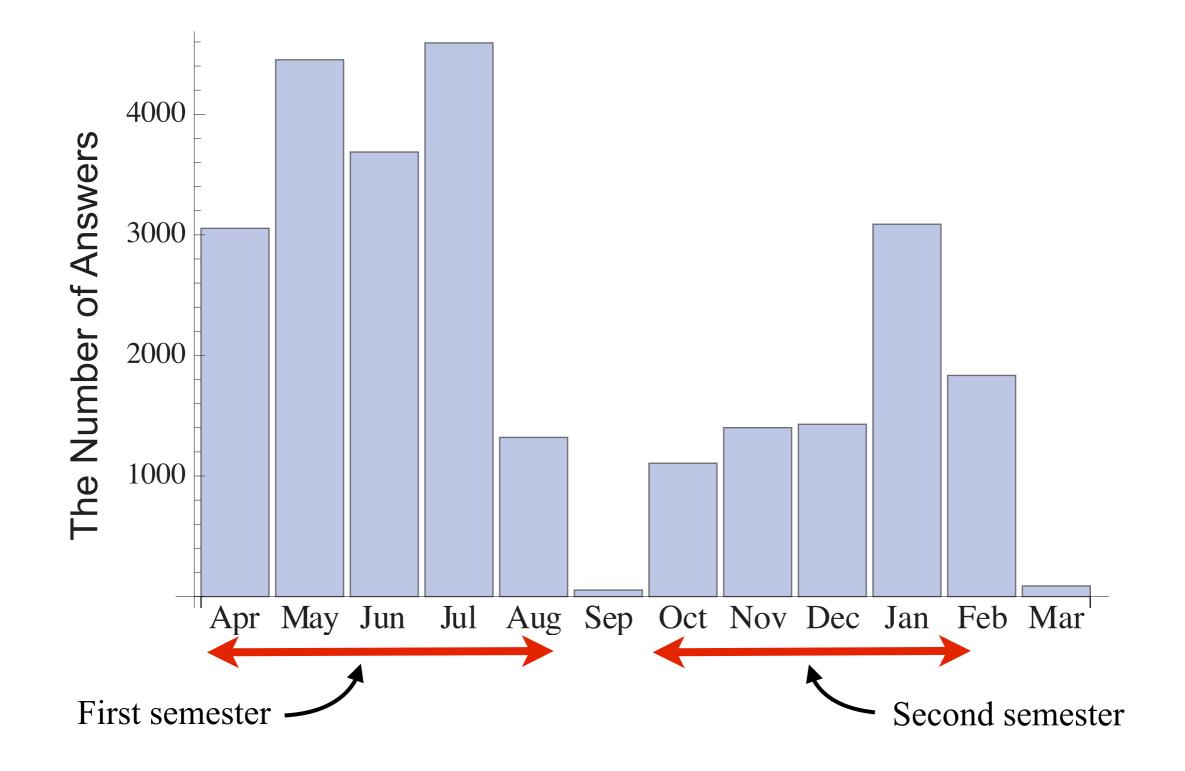
Effectiveness - Log analysis -



The number of users (WMLS and WASM)

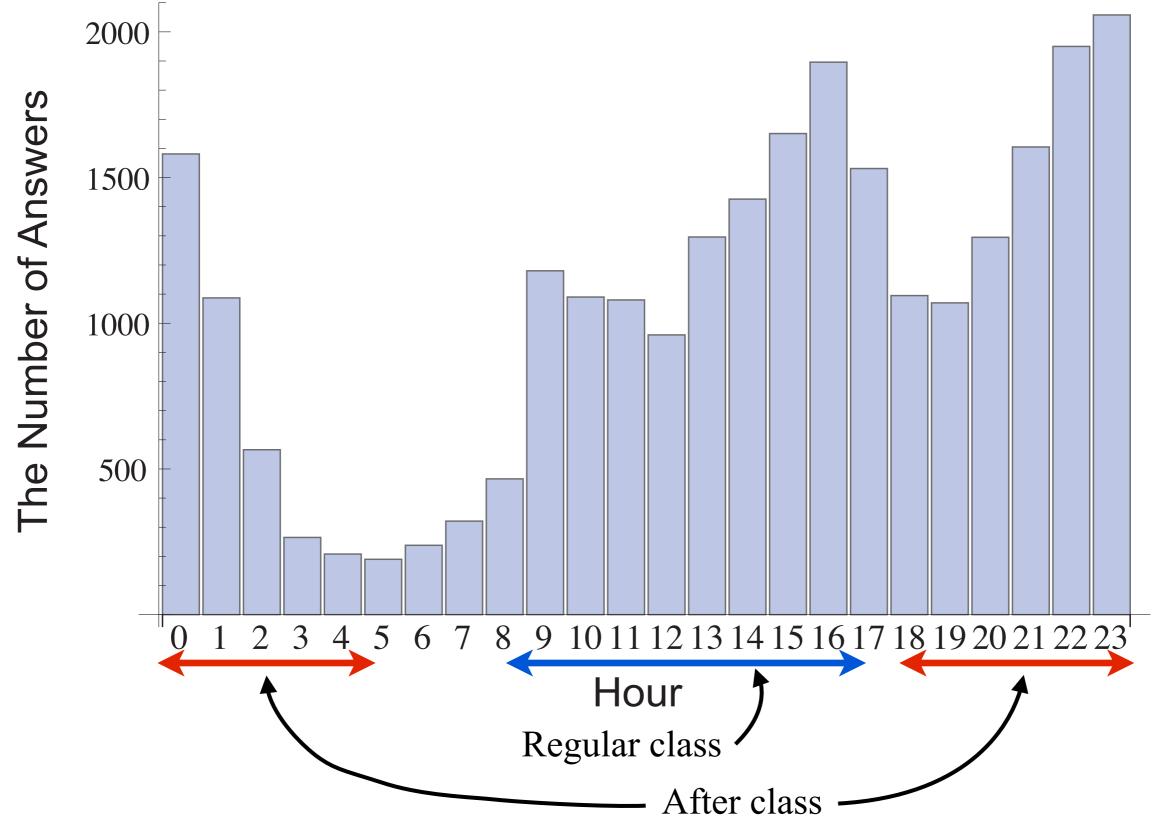


Frequency of use split by month (WMLS)



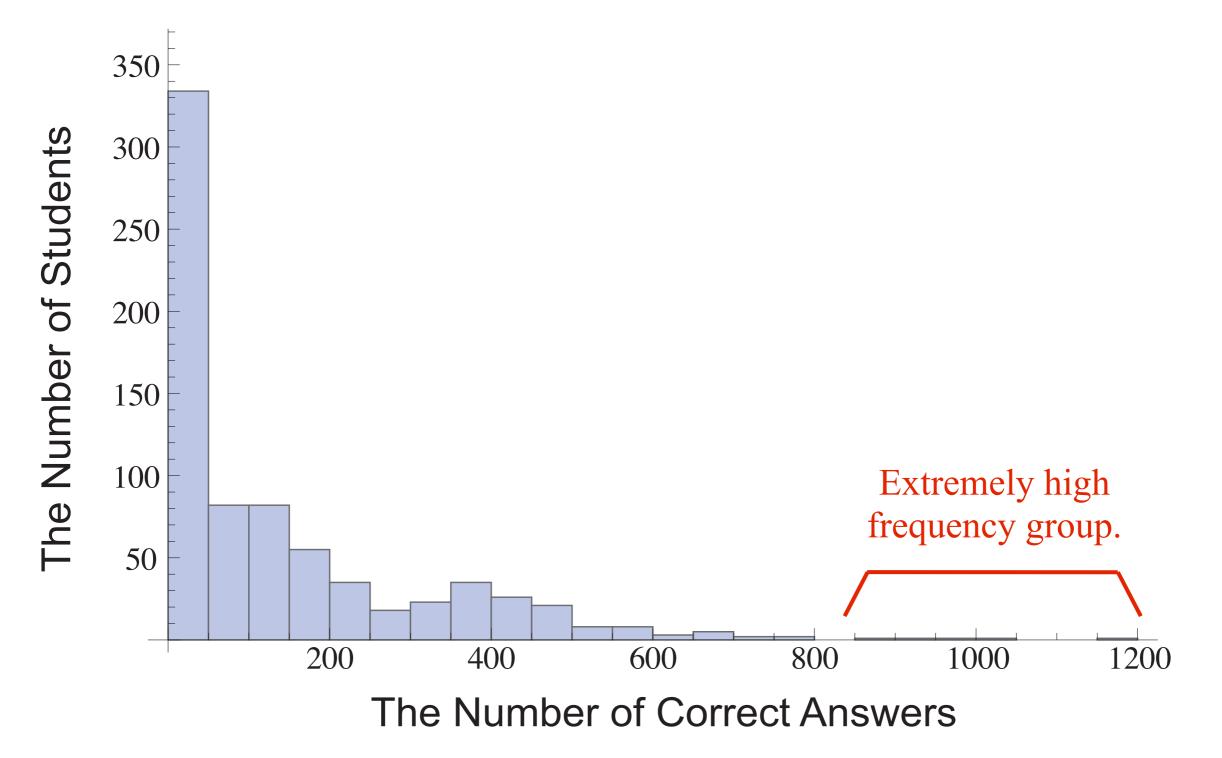
Frequency of use split by hour(WMLS)

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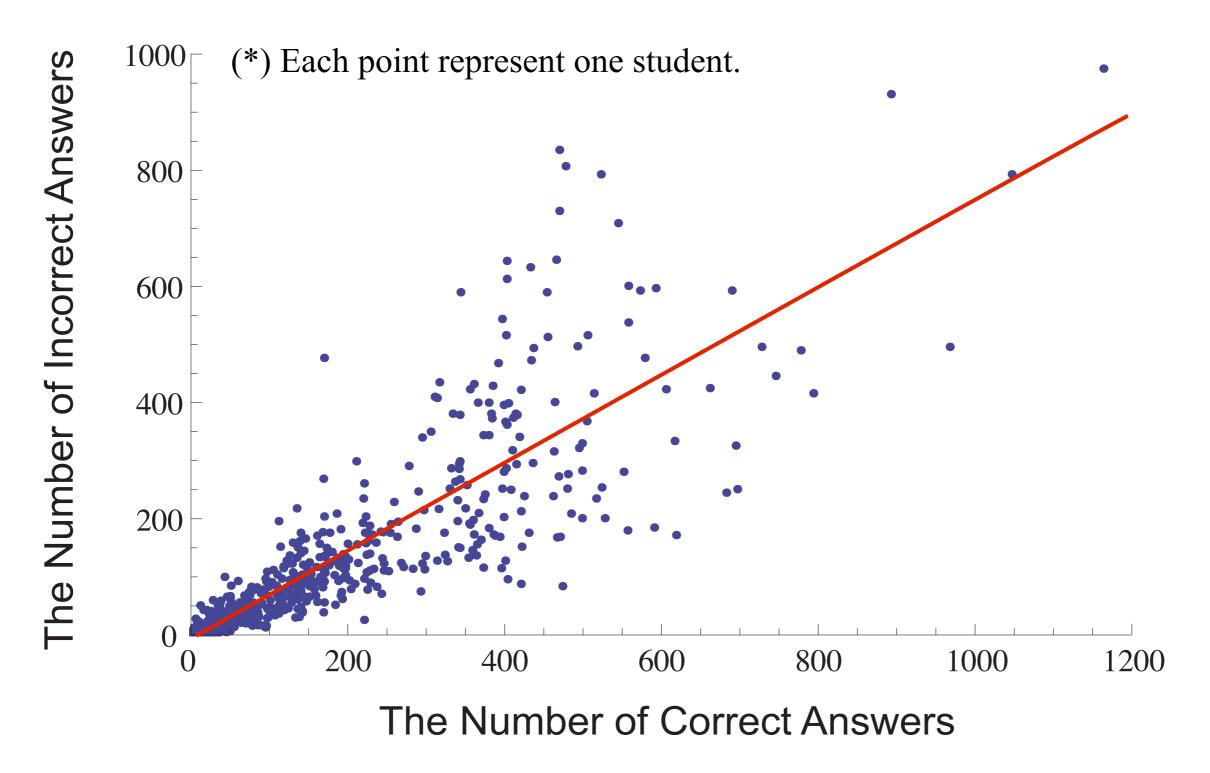


¹⁴ September 2016 @ Newcastle University

The number of students and correct answers (WMLS)



^{E-A×M-S} The number of correct and incorrect answers (WMLS)

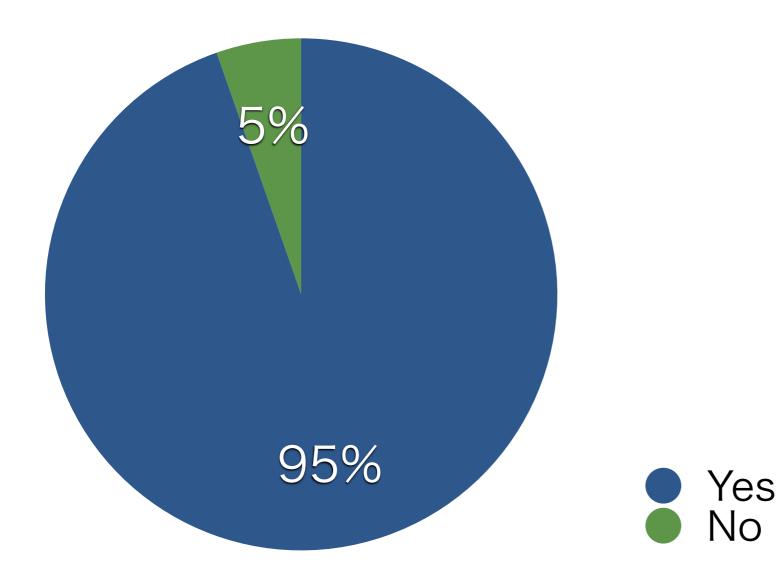


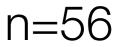
Effectiveness - Questionnaire Survey on WASM -(AY 2014, Fall)



Questionnaire survey on WASM (1) - Linear Algebra course / AY2014 Fall -

Have you ever used MASM?



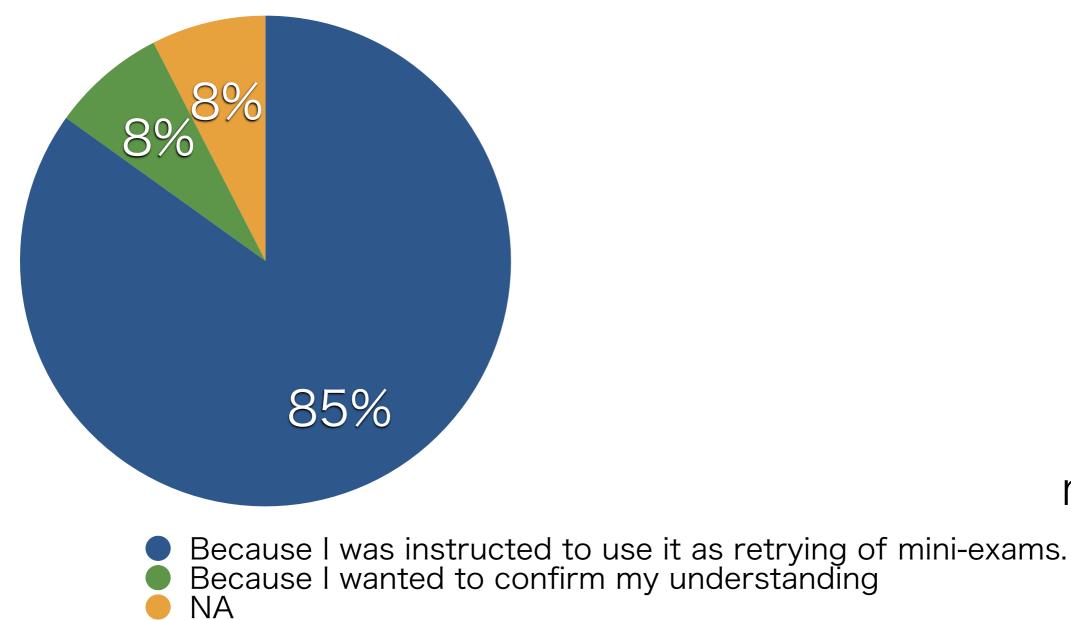




Questionnaire survey on WASM (2)

- Linear Algebra course / AY2014 Fall -For whom replied YES in the question (1)

Why did you use the WASM?



S

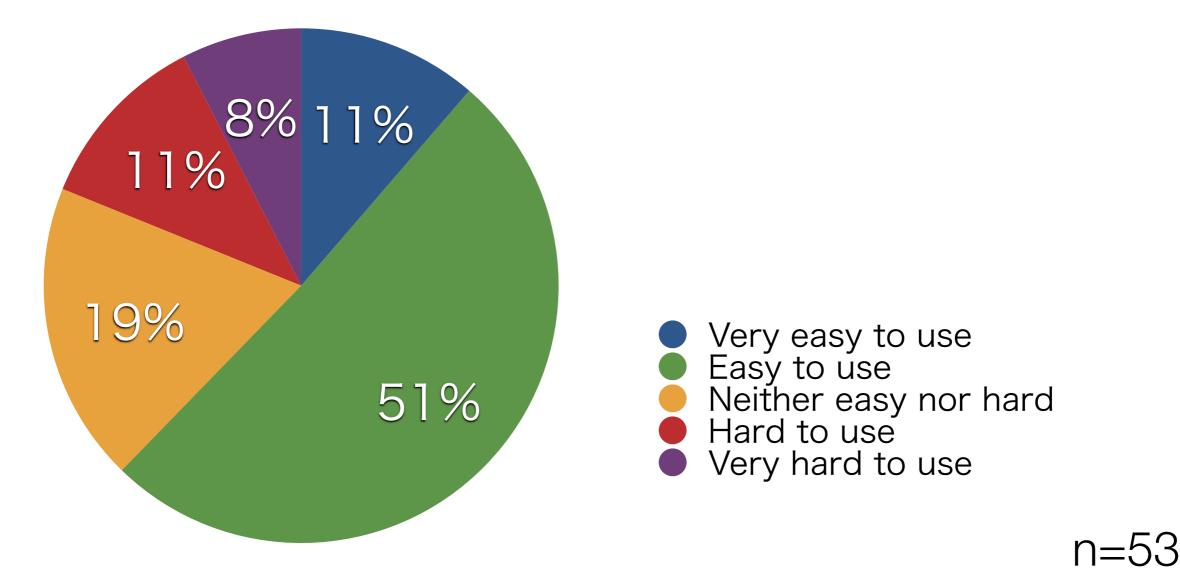
n=53



Questionnaire survey on WASM (3)

- Linear Algebra course / AY2014 Fall -For whom replied YES in the question (1)

What do you think about the usability of the system?

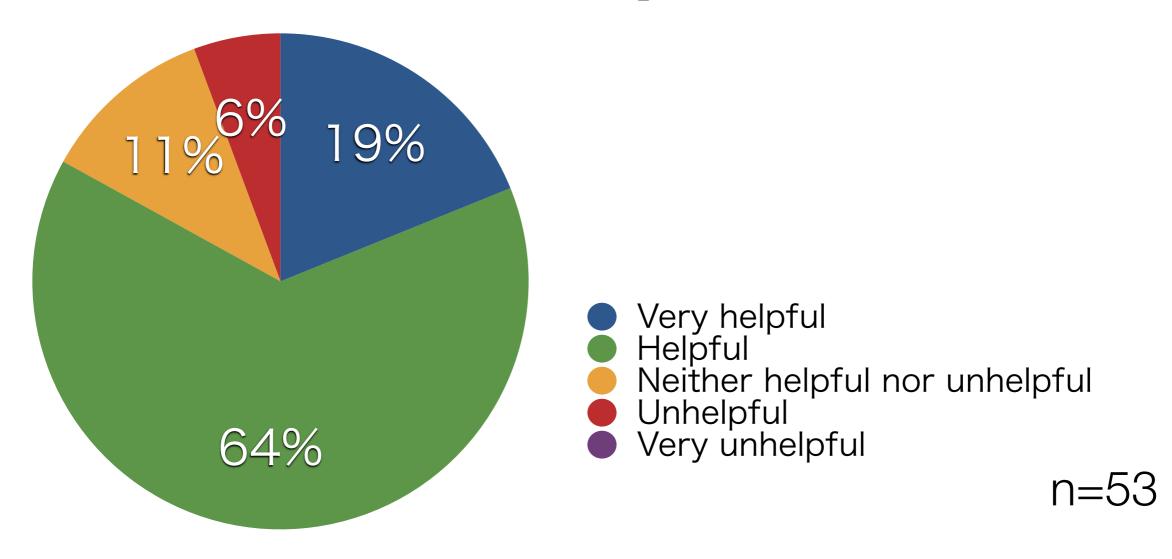




Questionnaire survey on WASM (4)

- Linear Algebra course / AY2014 Fall -For whom replied YES in the question (1)

Do you think the system is helpful for mastering computational procedures and checking your understanding of mathematical concepts?



Data structure



• Parameters ... List of fixed or programmed (random) parameters. The system decides randomly which parameter is used.

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- Question text and answer form ... Html based text (JSP). Parameters formatted by MSPFormat command (Image or MathJax)
- Judgement program ... Mathematica program using parameters and inputs. Any style is available and return a code. Typically Which[] is used. (first match semantic)
- Feedback message ... Consist of CSV of the codes and messages. Codes are the return value above.
- Example question and how to solve it ... PDF files are linked. To explain how to solve the problem.

Contents structures (overview via screenshot)

2 問題欄(*)	問題文:開始 問題表示のための前処理			
(100)で100(18)	<rpan style="font-size:18px;">次の定積分を計算せよ. <msp:evaluate></msp:evaluate></rpan>			
アップロード: 選択 ファイルカ	<pre>stF = mondai[[2]]; st = ToExpression[stF]; edF = mondai[[3]]; ed = ToExpression[edF];</pre>			
-	95 El 3 -4-10. P	教々 (市教女/オ) (1)(1)(1)(1)(1)(1)(1)(1)(1)(1)(1)(1)(1)(
解答標(*) 別ックドゥで編集 アップロード: 満れ ファイルも 確認真面編集	if (typed \$("#tbl") /nokbd-icon \$("form :) {// 2013.08.05a f \$.keypad != "undefined") { .after(' <img alt="Keypad:Off" height="30px;" id="kpf" src="image/common
.jpg"/> '); text").keypad('destroy'); ptElog = 'Off'.		
2				
解答欄チェック (*) 別ッのドッで編集 マップロード:	FullSimp	Null, mrk=0, lify[ans - seikai]0, mrk=1, k=2;ecd=1	D	
	(3847)			
2		ルが選択されていません。 ルが選択されていません。		
画像ファイル	選択 ファイルが選択されていません。 登録画像一覧 画像パスコピー 選択 ファイルが選択されていません。 選択 ファイルが選択されていません。			
	反政	{a = RandomChoice[{1/2, 1/3, 2/3}]; b = RandomChoice[{1/2, 1/3, 2/3}];		
問題パラメータ	mondai	<pre>tandomchoice[[1/2, 1/3, 2/3]]; b = Randomchoice[[1/2, 1/3, 2/3]]; ToString[Sin[a*x] + Cos[b*x], InputForm], "0", "Pi/2"}</pre>		
アップロード:		る場合は、入力後、必ず[追加]ボタンをクリックしてください。↓↓↓		
選択 ファイルカ			20	
		اد ملی د محمد ا		
解答エラー	0		N.IR	
メッセージ	1		机酸	
アップロード: 実祝ファイルカ		る場合は、入力後、必ず[追加]ホタンをクリックしてください。↓↓↓	18.20	
2	必須 判定変	2011年1月1日日日日日日日日日日日日日日日日日日日日日日日日日日日日日日日日日		
変数	Mrk 🖉	ans ans Ang数表示		
アップロード:	※↓↓↓追加する場合は、入力後、必ず[追加]ボタンをクリックしてください。↓↓↓ 入力変数表示			
選択 ファイルカ		3810	- I	
クリア基準(%)(*)	100			
例題と解説	選択 ファイルが選択されていません。			
ログ(*)	{mondai, ans, mrk, ecd} ログ内容転記			

- \leftarrow Question sentence
- ← Answer form
- ← Judgement program

- ← Parameters
- ← Feedback messages



Contents samples (question text)

```
<msp:evaluate>
mon=mondai[[1]]; MCT=Length[mon];
seikai=Array[0,MCT]; hf=Array["",MCT];exp=Array["",MCT];
For[i=1,i \le MCT,i=i+1,seikai[[i]] = ToExpression[mon[[i]]];hf[[i]] = "HoldForm[" <> mon[[i]] <> "]";
 exp[[i]] = StyleForm[ToExpression[ hf][i]] ], FontSize->18]];
pi = StyleForm[Pi, FontSize->18];
</msp:evaluate>
Find the next values.
(1)
\(\displaystyle\large<msp:evaluate>ToString[TeXForm[exp[[1]]]]</msp:evaluate>\)</div>
(2)
\(\displaystyle\large<msp:evaluate>ToString[TeXForm[exp[[2]]]]</msp:evaluate>\)</div>
(3)
\(\displaystyle\large<msp:evaluate>ToString[TeXForm[exp[[3]]]]</msp:evaluate>\)</div>
(4)
\(\displaystyle\large<msp:evaluate>ToString[TeXForm[exp[[4]]]]</msp:evaluate>\)</div>
(5)
\(\displaystyle\large<msp:evaluate>ToString[TeXForm[exp[[5]]]]</msp:evaluate>\)</div>
(6)
\(\displaystyle\large<msp:evaluate>ToString[TeXForm[exp[[6]]]]</msp:evaluate>\)</div>
(7)
\(\displaystyle\large<msp:evaluate>ToString[TeXForm[exp[[7]]]]</msp:evaluate>\)</div>
(8)
\(\displaystyle\large<msp:evaluate>ToString[TeXForm[exp[[8]]]]</msp:evaluate>\)</div>
14 September 2016 @ Newcastle University
```



Contents samples (answer form)

```
Answer form
 <input type="text" name="cx" value="<msp:evaluate> MSPValue[ $$cx, ""] </msp:evaluate>" size="3"
autocomplete="off" onkeyup="javascript:checkInput(this.form.cx);">
  x + 
 <input type="text" name="cy" value="<msp:evaluate> MSPValue[ $$cy, ""] </msp:evaluate>" size="3"
autocomplete="off" onkeyup="javascript:checkInput(this.form.cy);">
 y+
 <input type="text" name="cz" value="<msp:evaluate> MSPValue[ $$cz, ""] </msp:evaluate>" size="3"
autocomplete="off" onkeyup="javascript:checkInput(this.form.cz);">
  z + 
 <input type="text" name="cc" value="<msp:evaluate> MSPValue[ $$cc, ""] </msp:evaluate>" size="3"
autocomplete="off" onkeyup="javascript:checkInput(this.form.cc);">
 =0
 <font color=red>
In the case that coefficients are 0 or 1, enter 0 or 1, and in the case that answer is negative one, enter like as (-2) or -2.
</font>
```



Contents samples (parameters)

Simple Case

mondai, {"ArcTan[Sqrt[3]/2]+ArcTan[Sqrt[3]/5]", Pi/3} mondai, {"ArcSin[11/14]+ArcSin[13/14]", 2Pi/3} mondai, {"ArcSin[1/3]+2ArcSin[1/Sqrt[3]]", Pi/2} mondai, {"ArcTan[3/4]+ArcSin[4/5]", Pi/2}

Random Case

 $mondai, \{p=RandomInteger[3]+2;1/(n(n+p))\} \\ mondai, \{p=RandomInteger[3]+2;q=RandomInteger[2]+1;1/Expand[(p*n+q)*(p*(n+1)+q)]\} \\ mondai, \{a=RandomInteger[3]+1;b=(RandomInteger[5]+1)/(RandomInteger[]+1);(a*n+2*b)/(n(n+1)(n+2))\} \\ mondai, \{a=RandomInteger[3]+1;b=(RandomInteger[5]+1)/(RandomInteger[]+1);(a*n+2*b)/((n+1)(n+2)(n+3))\} \\ \label{eq:stars}$

Complex case (programmed)

 $\begin{array}{l} mondai, \{a=RandomInteger[3]+2;"(-1/" <> ToString[a] <> \\ ")^n", p=RandomInteger[3]+1; q=RandomInteger[3]+1; r=RandomInteger[3]+2;"(" <> If[p>1, ToString[p],""] <> "n+" <> \\ ToString[q] <> ")/" <> ToString[r] <> "^n" \} \\ mondai, \{a1=RandomInteger[4]+4; b1=RandomInteger[a1-2]+1; b=Numerator[b1/a1]; a=Denominator[b1/a1];"(" <> \\ ToString[b] <> "/" <> ToString[a] \\<>)^n", p=RandomInteger[3]+1; q=RandomInteger[3]+1; r=RandomInteger[3]+2;"(-1)^(n-1)(" <> If[p>1, ToString[p],""] \\<> "n+" <> ToString[q] <> ")/" <> ToString[r] <> "^n" \} \\ \end{array}$

* "mondai" means problem in English.



Contents samples [judgement] (1)

```
ecd=0;sel=ToExpression[sel];
BadPat=RegularExpression[".*Limit.*|.*->.*"];
Which[
!MemberQ[{0,1,2,9},sel], mrk=0;ecd=1,
sel===0 && val===Null, mrk=0;ecd=2,
sel===0 && StringMatchQ[ansstr,BadPat]===True,mrk=2;ecd=5,
sel===0 && NumericQ[val]===False, mrk=2;ecd=5,
asel=!=sel,mrk=2;ecd=3,
asel===sel && asel===0 && aval=!=val,mrk=2;ecd=4,
asel===sel && asel===0 && aval===val,mrk=1,
asel===sel && asel=!=0,mrk=1,
True, mrk=2;ecd=99
];
```



Contents samples [judgement] (2)

```
ecd=0;
Which[
matq===Table[Null, {i,1,4}, {j,1,4}] && matd===Table[Null, {i,1,4}, {j,1,4}], mrk=0;ecd=1,
MemberQ[Flatten[{matq,matd}],Null], mrk=0;ecd=2,
Simplify[matq.Transpose[matq]]=!=IdentityMatrix[4], mrk=2;ecd=3,
Simplify[mondai.matq-matq.matd]===Table[0, {i,1,4}, {j,1,4}], mrk=1,
True, mrk=2;ecd=4
```

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Contents samples [judgement] (3)

```
ecd=0;ans={ans1,ans2,ans3,ans4};
PolynomialQx[fx] := Module[\{v\}, v = fx / x \rightarrow 1;
If[! NumericQ[v], Return[False]];
 If[! PolynomialQ[fx, x], Return[False]]; Return[True]];
TmpDeg[fx ]:=Exponent[fx,x]; AnsDegs=Map[TmpDeg,ans];
ChkDegs=\{\};For[i=1,i<=MCT,++i,
If[Exponent[ans[[i]],x] <=
n[[i]],ChkDegs=Append[ChkDegs,True],ChkDegs=Append[ChkDegs,False]]];
ChkPX=0;For[i=1,i<=MCT,++i,
 If [Limit[(FX[[i]]-ans[[i]])/x^n[[i]],x->0]!=0,++ChkPX]]
Which[
 MemberQ[ans,Null], mrk=0;ecd=1,
 MemberQ[Map[PolynomialQx,ans],False], mrk=2;ecd=2,
 MemberQ[ChkDegs,False], mrk=2;ecd=3,
 MemberQ[ChkPX,False], mrk=2;ecd=4,
 ChkPX > 0, mrk=2;ecd=5+ChkPX,
 ChkPX == 0, mrk=1,
 True, mrk=2;ecd=99
];
```



Moodle plugin



Problems of the system

- Usability for the teachers : difficult to handle the learning status.
- An problem caused by upgrade of OS occurred at early in 2015.

Fortunately our university provides the LMS (Moodle).



Moodle plugin (assisted by Nakahara)

Spec :

- Full compatible with WASM w.r.t. question data.
- Network license of Mathematica required.

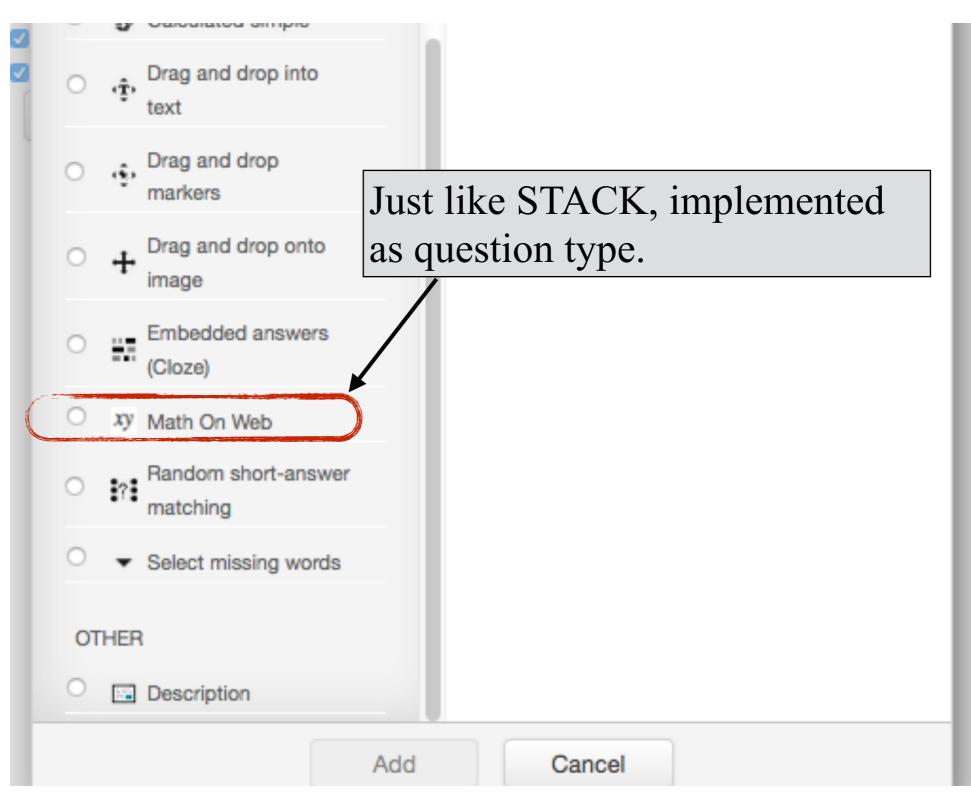
History and Plan :

- 2015 : Developed prototype (which can handle simple questions)
- 2016 : Bug fix.

Addition of function to import from WASM data archives. Other improvements('LTI' may be included?).

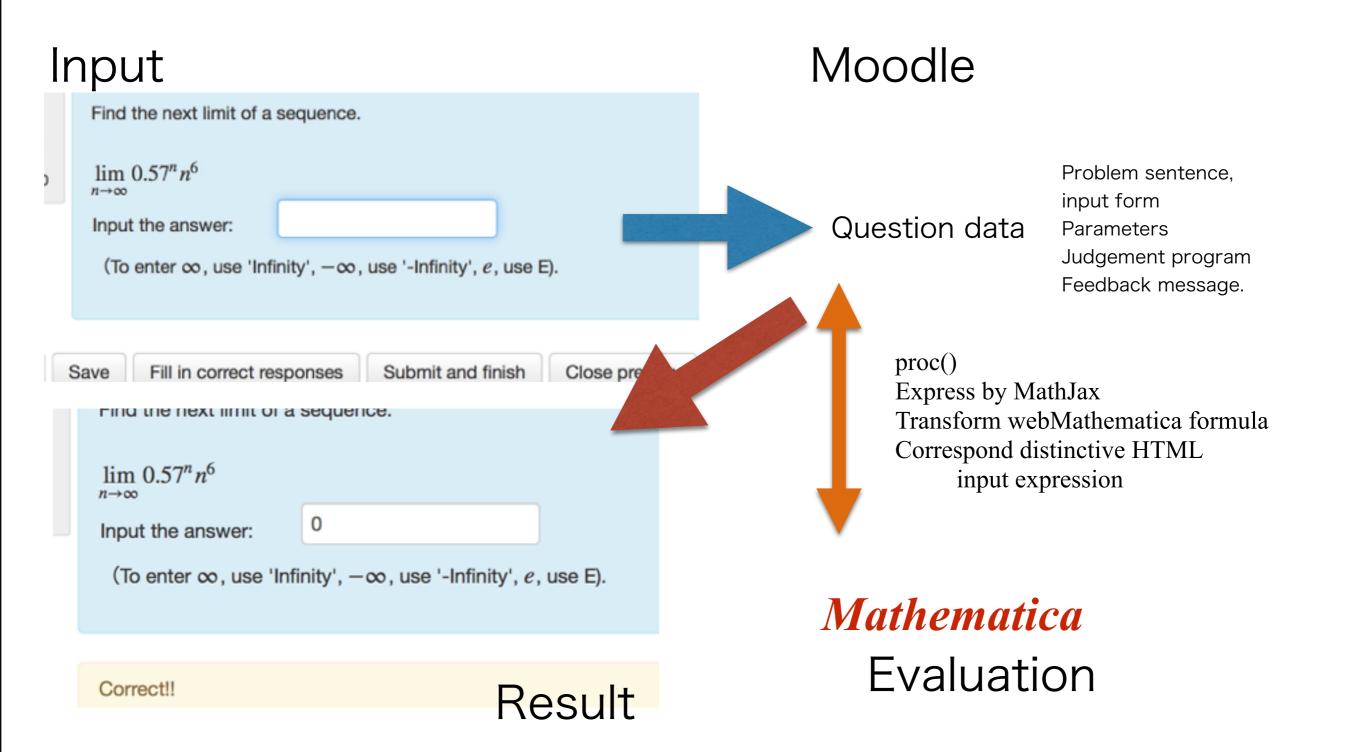
- 2017 : Starting test use.
- 2018 : Starting practical use.

Screenshots





System of the plugin



'MeLQS'

the project for sharing materials among heterogeneous systems



What we want to do

- We can use STACK 3.x in our university.
- We have already more than 1200 question data.
- BTW, there are so many drill or assessment systems. e.g. Maple T.A. Numbas, DEWIS,...
- The <u>question data</u> is the key of such systems and should be <u>shared.</u>
- But how? Converter ?

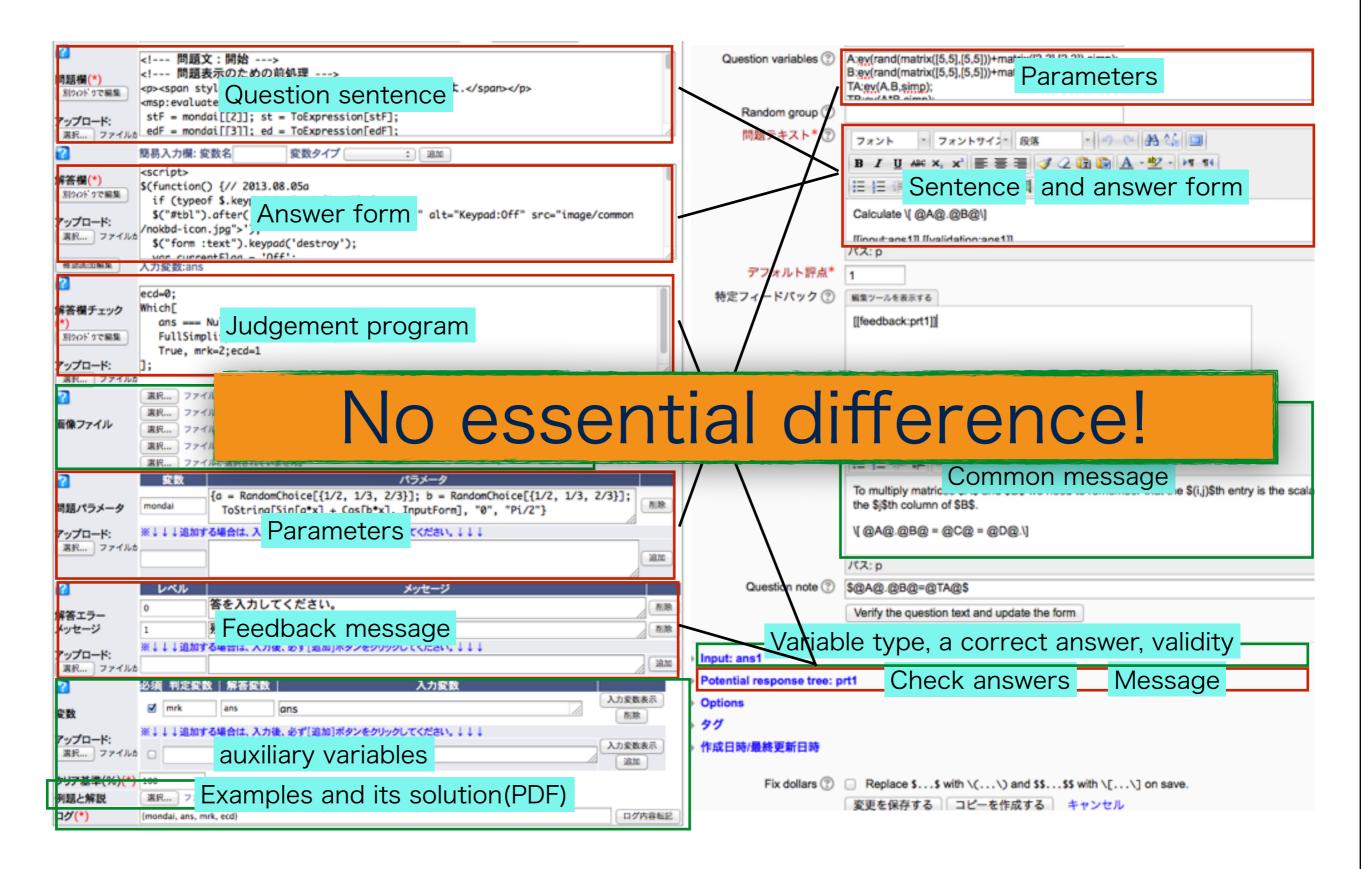


Structures of the systems

	WASM	STACK	Notes for mutual use of contents data
CAS	Mathematica	Maxima	Number and kind of functions are different
Contents Structure	Question sentence Parameters Answer form Judgement program Feedback message	Question sentence including answer form Quest. variable PRT Sample answer	A little different, but almost the same, correspondence is clear
Language	JSP(HTML)/MathJax Mathematica formula	HTML/MathJax CAS Text (maxima)	Basically same as difference of CAS, outputs are different
Misc	Judgement only, no correct answer required	display a correct answer.	STACK is assumed to be prepared a correct answer.
	Code is full compatible with CAS(subset).	Supported command not fully compatible with CAS	Feedback variables fit for PRT should be computed.



Compare the contents data





F-A×M+S

- Question data formats may be quite analogous even if the systems are different.
- We want to share question data among heterogeneous systems.
- Rather than a converter, it will be more useful to store the question data in accordance with more generic format.

'Specification' is the key of question data.

We call this "Mathematics e-Learning Question Specification" or 'MeLQS' for short.



E-A×M+S 2016

- We have developed a *Mathematica*-based drill and assessment systems WMLS and WASM to create a blended learning environment.
- We have more than 1200 questions. We think the question data is the most important element of such systems and will be glad to disclose the data to the public, in a compatible format, like as XML.
- Aiming to share the contents data among heterogeneous systems, we suggested "Mathematics e-Learning Question <u>Specification</u>", which we call 'MeLQS' for short ([2014Y]).
- We are starting to survey how the '<u>Specification</u>' should be, cooperating with Japanese STACK contributors, Y. Nakamura and T. Nakahara, and an Application Engineer for Maple T.A., K. Kato.

[2014Y] K.Yoshitomi, On a formulation of "Mathematics e-Learning Contents Specification" and it's applications to some systems, The 39th Annual Conference of JSiSE (Japanese Society for Information and Systems in Education) at Wakayama Pref.



Finally...

- By sharing the question data, all of teachers will be so happy.
- Even if they use any other system, they can concentrate to improve their class using questions shared and imported from MeLQS db.

If you are interested in the question data or MeLQS, please feel free to contact us below :

yositomi@las.osakafu-u.ac.jp