Using STACK to Assess Information Transfer in Mathematics

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Outline

1. Online and blended math courses at UH
2. Mathematical competencies and information transfer, extreme apprenticeship method
3. Assessing information transfer with STACK

Online Calculus Courses

- Single Variable Calculus 2004
- Calculus I & II
- Advanced Calculus 2013
  Completeness property of real numbers with ε - δ rigorous proof method
- Option to take University of Helsinki Calculus curriculum completely online and in English starting Fall 2015

+Introduction to Logic

Ordering and the Completeness of Real Numbers

This lecture introduces the properties of the operations of real numbers (addition, subtraction, multiplication, and division). Ordering of numbers and the **Completenes**[s of] the set of real numbers are discussed.

- Operations, Ordering, and the Completeness of Real Numbers
  - Handout
  - Slides
  - Slideshow
- Solved Problems on Operations and Ordering
  - Handout
  - Slides
  - Slideshow
- Expressions
- Equations
- Ordering of Numbers

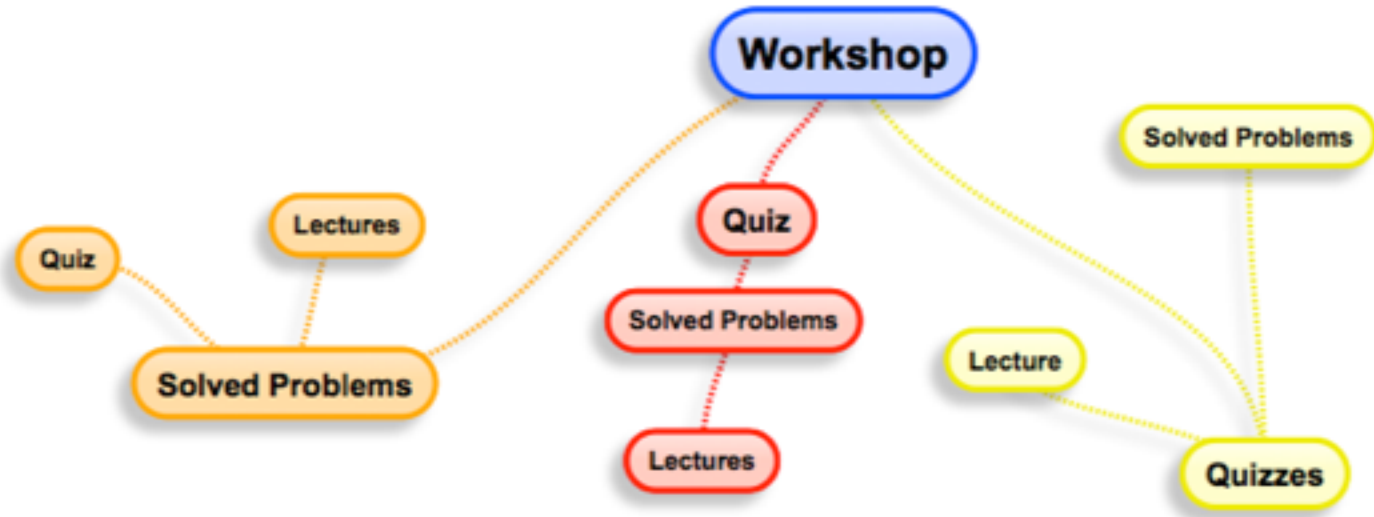
The following workshops requires students to write a proof, and then to assess other students' submissions

- Ordering x positive implies -x negative
- Ordering 1 > 0

Each module contains

- YouTube videos
- Reading materials
- **Quizzes (Automatic assessment)**
- **Workshops (Peer assessment)**
- Also: discussion forums, calendar, gradebook, etc.

Course Design



Supports individual study paths

Peer Graded Workshops

given homework problems asking a complete and clear presentation of a

By Wednesday students **submit their solutions**

solution together with assessment criteria are made

grades and gives **constructive feedback** to 4-5 other students'

published. A student's grade is the average of the given grades. **Instructor**

Blended Courses at UH

- **Linear Algebra and Matrices**
- Introduction to University Mathematics
- High school math refresher with Abacus
- Multivariate Calculus

+Statistics courses

- Part of homework returned by paper, part to automatic or peer assessment
- Lectures and tutoring sessions

Extreme Apprenticeship Teaching Method

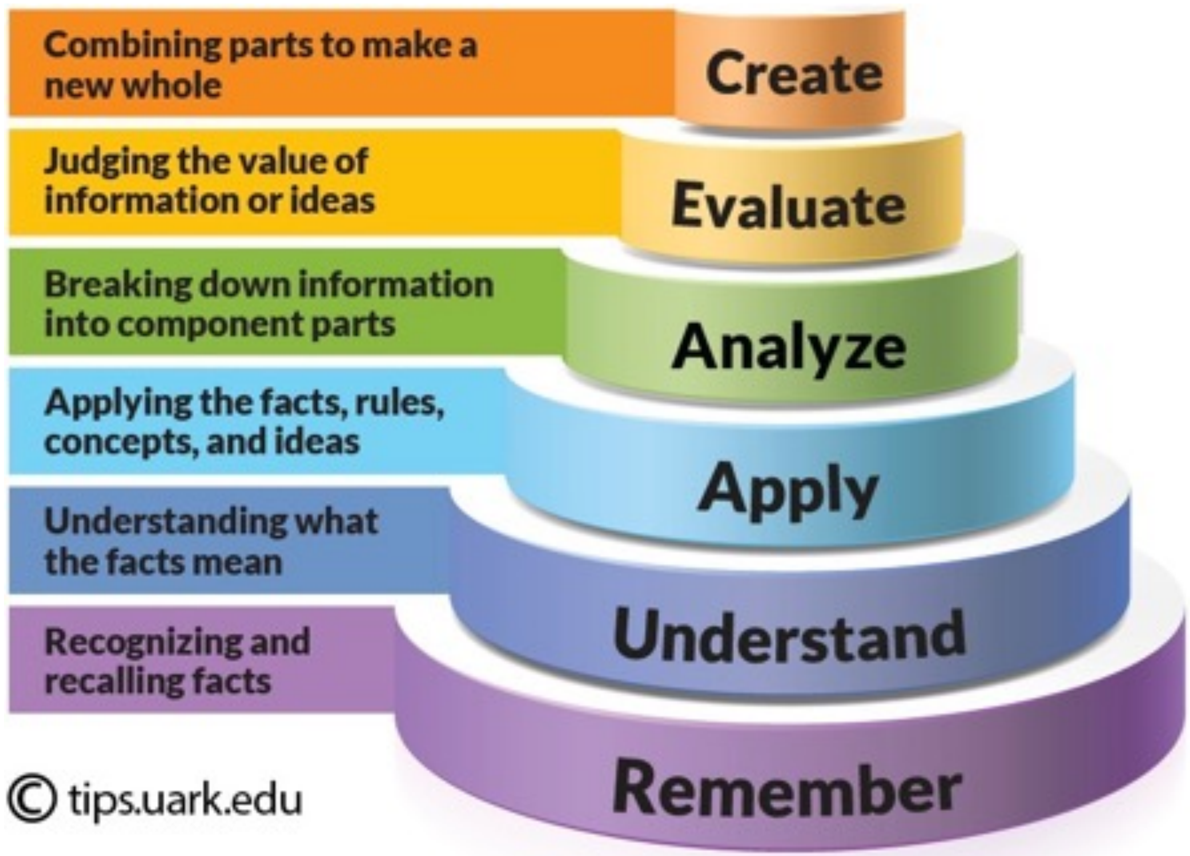
- Amount of lectures significantly smaller
- Amount of tasks is substantially larger than traditionally, approximately 15–20 problems per week
- Assignments force students to investigate topics by reading the course material prior to the lectures
- Part of the tasks are marked by teaching

Mathematical Competencies

(Pointon & Sangwin 2003, Rämö, Oinonen, Wikberg 2015)

1. Factual recall
2. Carry out a routine calculation or algorithm
3. Classify some mathematical object
4. Interpret situation or answer
5. Proof, show, justify (general argument)
6. Extend a concept
7. Construct example/instance
8. Criticize a fallacy
9. **Information transfer**

Bloom’s Taxonomy



Information Transfer

Questions that "require transformation of information from one form to another, as well as processing this information". This involves, e.g. drawing a picture of a given situation, interpreting, and explaining.

Examples of category 9:

- Denote $v_1 = (-3, 4)$, $v_2 = (1, 1)$ and $v_3 = (\frac{2}{3}, -2)$. Draw pictures of the subspaces $span(v_1)$, $span(v_1, v_2)$ and $span(v_1, v_3)$. You do not need to justify your answer.

- The vector space $\mathbb{R}^2$ has a basis $B = ((1, 1), (2, 3))$. Determine, by drawing a picture, a vector $u \in \mathbb{R}^2$ whose coordinates with respect B to are 3 and -2.

- Explain in your own words why an elementary matrix always has an inverse matrix.

Automatic Assessment

- Routine tasks need to be learned
- Students do almost all of the automatic tasks
- Higher categories
- Information transfer
- Tasks broken into subtasks
- Feedback according to mathematical properties of the solution
- JSXGraph, GeoGebra

Consider the vectors $\vec{v}_1 = (1,0,2), \vec{v}_2 = (0,2,2)$ and $\vec{v}_3 = (1,2,4)$. Tidy question | Question tests & deployed versions

Let the vector $\vec{w} = (w_1, w_2, w_3)$ be an arbitrary vector of the space $\mathbb{R}^3$. We want to find out if the vectors $\vec{v}_1, \vec{v}_2$ ja $\vec{v}_3$ span the vector space $\mathbb{R}^3$.

a)

What kind of equation should we examine?

1. $\vec{v}_1 + \vec{v}_2 + \vec{v}_3 = \vec{w}$
2. $x_1\vec{v}_1 + x_2\vec{v}_2 + x_3\vec{v}_3 = \vec{w}$, where x_1, x_2 ja x_3 are real numbers
3. $\vec{v}_1 + \vec{v}_2 + \vec{v}_3 = \vec{0}$
4. $x_1\vec{v}_1 + x_2\vec{v}_2 + x_3\vec{v}_3 = \vec{0}$, where x_1, x_2 ja x_3) are real numbers
5. $\vec{v}_1 + \vec{v}_2 + \vec{v}_3 + \vec{w} = \vec{0}$
6. $x_1\vec{v}_1 + x_2\vec{v}_2 + x_3\vec{v}_3 + \vec{w} = \vec{0}$, where x_1, x_2 ja x_3 are real numbers

Enter the number of your choice in to the answer field below.

b)

Form a matrix that corresponds to the equation you selected. You can enter the vector components represented by letters by entering the index of the component directly after the letter. For example the vector $\vec{w}$ has the component w_1, which can be entered to the answer field by writing *w1*.

$$\begin{bmatrix} 1 & 0 & 1 & w1 \\ 0 & 2 & 2 & w2 \\ 2 & 2 & 4 & w3 \end{bmatrix}$$

c)

Transform the matrix in to row echelon form. Write the indexes of vector components directly after letters. For example $w_1 - 2 \cdot w_2$ is entered in to the answer field by writing *w1-2*w2*.

Answers:

$$\begin{bmatrix} 1 & 0 & 1 & \text{w1} \\ 0 & 2 & 2 & \text{w2} \\ 0 & 0 & 0 & \text{2*w3-2*w2-4*w1} \end{bmatrix}$$

d)

Does the equation examined in a) have answers when the vector $\vec{w}$ is an arbitrary vector of the space $\mathbb{R}^3$?

1. Yes.
2. No.

Answer: 2

e)

Is the following proposition true: The vector $\vec{w}$ is a member of the subspace spanned by the vectors $\vec{v}_1$, $\vec{v}_2$ and $\vec{v}_3$? How can you justify your answer?

1. The proposition is true.
2. The proposition is false.

Answer:

f)

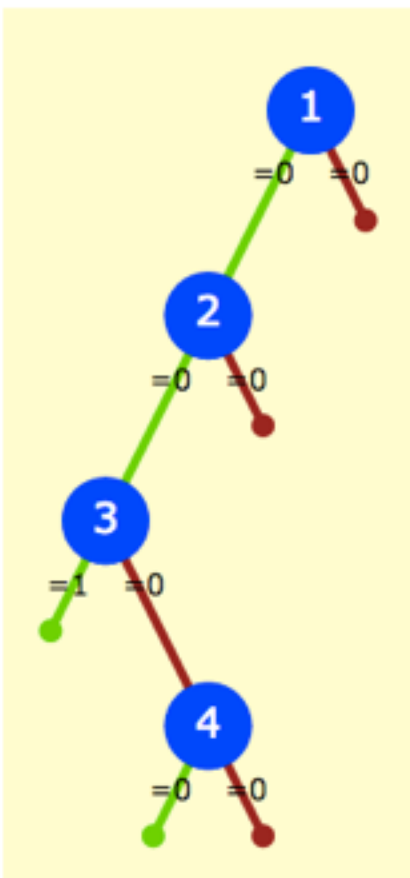
Is the following proposition true: The vectors $\vec{v}_1$, $\vec{v}_2$ and $\vec{v}_3$ span the space $\mathbb{R}^3$? How can you justify your answer?

1. The proposition is true.
2. The proposition is false.

Feedback variables

```
subRAns2:submatrix(rAns2, matrix_size(ans2)[2]);
subRPorrasmuoto:submatrix(rPorrasmuoto, matrix_size(rPorrasmuoto)[2]);
```

This potential response tree will become active when the student has answered: **ans2**



Solution

a)

If a group of vectors span a vector space then any vector of that vector space has to be able to be written as a linear combination of the spanning vectors. Thus if $\vec{w}$ is an arbitrary vector of the vector space $\mathbb{R}^3$, we are interested in equation 2:

$$x_1\vec{v}_1 + x_2\vec{v}_2 + x_3\vec{v}_3 = \vec{w}.$$

b)

The matrix corresponding to the equation is formed by writing the factors of the vectors $\vec{v}_1, \vec{v}_2$ ja $\vec{v}_3$ so that each vector forms one column. The components t w_1, w_2 ja w_3 of the vector $\vec{w}$ are entered in the rightmost column.

When forming a matrix by had its good to note that the number of rows in the matrix has to be the same as the number of components in the vectors. The number of columns in the matrix has to be the same as the number of vectors in the equation.

The matrix corresponding to the equation is:

$$\begin{bmatrix} 1 & 0 & 1 & w_1 \\ 1 & -1 & 0 & w_2 \\ 2 & 3 & 5 & w_3 \end{bmatrix}.$$

c)

The matrix is transformed in to row echelon form by performing the necessary elementary row operations. These are:

- Multiplying a row with a positive or negative real number.
- Adding a row to another row multiplied by a real number.
- Switching two rows.

Its easy to make mistakes while performing elementary row operations. A practical way of avoiding mistakes is to do just one operation at a time.

Example from Logic Course

Let $L = \{P, R\}$ be a vocabulary, where P is a unary predicate symbol and R a binary predicate symbol. Let M be the set $\{1, 2, \ldots, 9\}$ (whole numbers from 1 to 9). Let us define two L-structures $\mathcal{M}_1$ and $\mathcal{M}_2$ as follows:

Tidy question | Question tests & deployed versions

The universe of both $\mathcal{M}_1$ and $\mathcal{M}_2$ is the set M.

$P^{\mathcal{M}_1} = \{1, 2, 3\}$ and $P^{\mathcal{M}_2} = \{7, 8, 9\}$.

$R^{\mathcal{M}_1} = R^{\mathcal{M}_2} = \{(1,4), (4,1), (1,7), (7,1), (7,4), (4,7)\}$.

a) Define a function f that fulfills the following conditions:

1° f maps all elements of the universe of $\mathcal{M}_1$ to elements of the universe of $\mathcal{M}_2$.

2° Every element of the universe of $\mathcal{M}_2$ is the image of exactly one element of the universe of $\mathcal{M}_1$.

3° If a, b and c are in the universe of $\mathcal{M}_1$, then

$a \in P^{\mathcal{M}_1}$ if and only if $f(a) \in P^{\mathcal{M}_2}$ and

$(b, c) \in R^{\mathcal{M}_1}$ if and only if $(f(b), f(c)) \in R^{\mathcal{M}_2}$

$$f(x) = \begin{cases} \square, & \text{if } x = 3 \\ \square, & \text{if } x = 4 \\ \square, & \text{if } x = 5 \\ \square, & \text{if } x = 6 \\ \square, & \text{if } x = 7 \\ \square, & \text{if } x = 8 \\ \square, & \text{if } x = 9 \end{cases}$$

b) Read the definition of the "general case" of an isomorphism from the course material.

Which of the conditions of an isomorphism (ISO1, ISO2, ISO3) does the function f fulfill? Keep in mind the definitions of $\mathcal{M}_1$ and $\mathcal{M}_2$ and the vocabulary L we gave above. Make sure you can justify your answer.

1. ISO1
2. ISO1, ISO2
3. ISO1, ISO3
4. ISO1, ISO2, ISO3
5. ISO2
6. ISO2, ISO3
7. ISO3

$\square$

c) Are the structures $\mathcal{M}_1 = (M, P^{\mathcal{M}_1}, R^{\mathcal{M}_1})$ and $\mathcal{M}_2 = (M, P^{\mathcal{M}_2}, R^{\mathcal{M}_2})$ isomorphic?

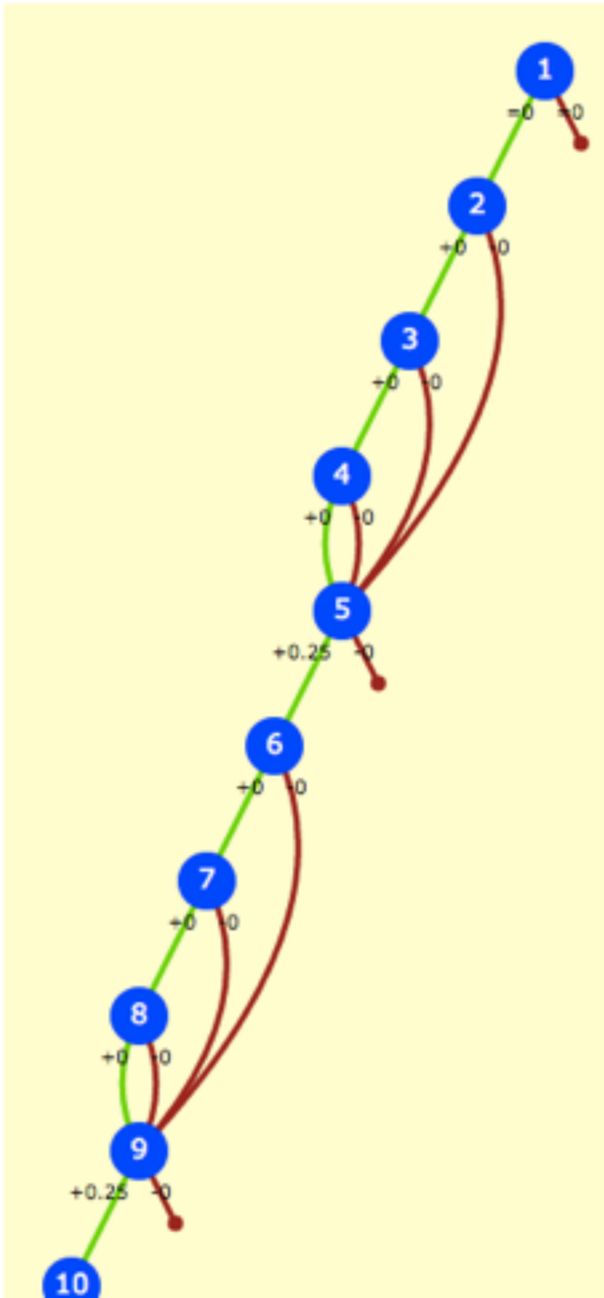
1. Yes.
2. No.

d) Let's take a minute to think about isomorphisms outside of this exercise. Assess your ability to prove whether two L-structures are isomorphic or not. We're assuming that the proof would be done with pencil and paper.

Pick the option that fits you the best and enter it in the text box below. There is no right or wrong answer.

1. I can prove or disprove isomorphisms for just about any L-structures (within reason) using the general-case-definition of an isomorphism.
2. I can prove or disprove some special cases, such as graphs, but might struggle with more general cases.
3. I would have some difficulties with both general and special cases without the use of the course material.
4. I don't quite understand the concept of an isomorphism very well yet and need to work on it a bit more.

This potential response tree will become active when the student has answered: **ans_a1, ans_a2, ans_a3, ans_a4, ans_a5, ans_a6, ans_a7, ans_a8, ans_a9**



"With help of the quiz, I finally understood isomorphisms"

Conclusions & Future work

- Meaningful and relevant feedback while working online
- Extending automatic assessment to cover various types of mathematical competencies
- Include dynamic graphics
- Automatic assessment of reasoning