DIAGNOSTIC TESTS IN NUMBAS

Don Shearman and Jim Pettigrew
Motivation/History

- Many students poorly prepared for university maths and stats study.
- Need for online tailored provision of refresher lessons in basic algebra.
- Need for an 'automated' system that diagnoses students' learning needs and recommends focus areas.
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- Need for online tailored provision of refresher lessons in basic algebra.
- Need for an 'automated' system that diagnoses students' learning needs and recommends focus areas.
- MESH adopted the Numbas Diagnostic Test algorithm.
- We received internal funding for development work.
Design and development

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The development involved refining the knowledge map and creating a set of Numbas questions.
DESIGN (KNOWLEDGE MAP)

The knowledge map has the form of a directed acyclic graph, where nodes represent topics (questions) and edges define the topic hierarchy.
DESIGN (DIAGNOSTIC)

The diagnostic tool was designed using Numbas.

- The Diagnostic exam type allows questions to be hierarchically arranged:
  - A wrong answer to a question causes the system to mark all harder questions on the same path as wrong.
  - A correct answer to a question causes the system to mark all easier questions on the same path as correct.
DEVELOPMENT

There we three key steps in the building process:

1. Matching questions to topics.
2. Creating learning outcomes and topics within them.
3. Linking topics according to the knowledge map (using 'depends on'/'leads to' directives).
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This development process was recursive in the sense that matching questions to topics led to some refinement of the knowledge map.
The Numbas authoring interface allows for easy topic linking:
Implementation

The tool was offered to students in February this year. To date (22/6/2023), it has been attempted 715 times by 521 unique students.

- Some attempts were incomplete, with some having no questions answered.
Analysis

Using attempt data from our LTI server, we have begun evaluating the diagnostic tool. Our main aim is to ensure that the knowledge map is valid and the questions based on it are properly constructed and targeted.

This is a work in progress (as determining a robust method of validation has required exploration of the research base).
Our analysis has unearthed a few unanticipated considerations:

- We have gained a deeper understanding of students' behaviour in interacting with the tool — for example:
  - some quit once they got a question (or a few questions) wrong.
  - some looked ahead and then quit.
  - many complete attempts took significant time (> 45 mins).
- There is scope for feature improvements of the tool — for example:
  - question sequence (perhaps a 'binary' selection?).
  - optimisation of efficiency of attempts: certain questions, if answered correctly, eliminate a large set of questions 'above' it.
DATA EXTRACTION, RESTRUCTURING, CLEANING

- Extraction via a JSON file.
- Restructuring and cleaning in R (dataframe with 'null' cases removed).
Knowledge map imported into R using the igraph package.

Each item is associated with 'out' and 'in' nodes — if a student gets a question (node) wrong, then the implication is that they will get all 'out' nodes wrong (the 'zeros sink'); if they get the question right, the implication is that they will get all 'in' nodes right (the 'ones float').
Algebraic Expressions

1.

101 - Order of operations

2.

102 - Write algebraic expressions

3.

106 - Expand brackets

4.

105 - Multiply and divide algebraic expressions

5.

104 - Collect like terms

6.

1.
IMPLIED SCORING

All non-NA raw question responses were converted to 0 or 1:

- response < 1 → response = 0
- response = 1 → response = 1

Knowing the sequence of a student's question responses, we have applied the above knowledge map logic to generate implied scores for all questions in the tool.

This has enabled the creation of 359 'fully implied scored' attempts (complete cases).
RASCH MODELLING

We have used Rasch modelling of raw and implied response sets (on complete cases only).

And compared the item difficulty rankings to glean information about questions and their place within the knowledge map hierarchy.

<table>
<thead>
<tr>
<th>Question Number</th>
<th>Item</th>
<th>N (raw)</th>
<th>M (raw)</th>
<th>xsi.item (raw)</th>
<th>RawRank</th>
<th>N (implied)</th>
<th>M (implied)</th>
<th>xsi.item (implied)</th>
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Q11 (205 - Fractions as division) was found to have the greatest difference in item difficult ranking (raw vs implied). For many cases, Q11 has been assigned an implied score of 1.
This might not be appropriate as Q11 contains a part that is relatively 'hard'.

<table>
<thead>
<tr>
<th>Q11: 205 - Fractions as division</th>
<th>Q15: 213 - Divide a number by a fraction</th>
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<tbody>
<tr>
<td>'Easy'</td>
<td>'Hard'</td>
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</table>
Q11: 205 - Fractions as division

Complete the following without using a calculator.

a) \( \frac{1}{3} \div \frac{1}{5} \)

Give your answer as a fraction (proper or improper).

Reduce your answer to lowest terms.

b) \( 1 - \frac{1}{3} \div \frac{6}{7} \)

Reduce your answer to lowest terms.

Q15: 213 - Divide a number by a fraction

Complete the following without using a calculator.

a) \( 12 \div \frac{1}{8} \)

Reduce your answer to lowest terms.

b) \( \frac{3}{17} \div 5 \)

Reduce your answer to lowest terms.